

**EXPLORING CHILDREN'S LEARNING OF NUMBER IN LOWER CLASSES
IN A MALAWIAN PRIMARY SCHOOL**

MASTER OF EDUCATION (PRIMARY) THESIS

By

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DECLARATION

I, the undersigned, hereby declare that this thesis is my own original work which has not been submitted to any other institution for similar purposes. Where other people's work has been used acknowledgements have been made.

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CERTIFICATE OF APPROVAL

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DEDICATION

This thesis is dedicated to my late beloved father who passed away in 2002 while I was pursuing a Diploma in Education. He always emphasised the value of education and encouraged me to work hard in school to reach as high as I could. I also dedicate the thesis to my beloved mother who always made sure that I had something to eat before going to school every morning. Finally, the thesis is dedicated to my beloved wife, Rhodys Chisoni, children: Grace, Brighton and Brandina for their support, understanding and enduring my absence over the period I had been studying.

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ABSTRACT

This thesis reports on an exploration of children's learning of number in lower classes in a Malawian primary school. The study used qualitative approach using a case study design and the sample included 5 mathematics teachers and 12 learners from Standards 1 and 2. Data was collected through lesson observations, semi-structured interviews, oral assessment interviews and documentary analysis. Analysis of the data collected was done thematically. Using a conceptual framework called Learning Framework in Number (LFIN); developed for the Mathematics Recovery (MR) Programme in 1990s in Australia, the study revealed that teachers teach early number concepts using counting strategies. The study revealed that children understand that adding and subtracting systematically increase and decrease the value of cardinality, even if they cannot reliably count the number of items involved. The study also found that learners use three types of strategies namely; counting, reasoning and mastery. Children's levels in five categories of basic numeracy were also determined. These were forward number word sequences (FNWSs), backward number word sequences (BNWSs), numeral identification (NI), number word after (NWA) and number word before (NWB). Generally, learners found FNWSs and NWA easier than BNWSs and NWB. The study made recommendations in the areas of teaching strategies and mathematics curricula in lower primary school.

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LIST OF ACRONYMS AND ABBREVIATIONS

BNWS	Backward Number Word Sequence
BTS	Base-Ten Strategies
CBCC	Community Based Childcare Centre
CGI	Cognitively Guided Instruction
ECD	Early Childhood Development
EGMA	Early Grade Mathematics Assessment
FNWS	Forward Number Word Sequence
FPE	Free Primary Education
LFIN	Learning Framework in Number
LiEP	Language in Education Policy
MIE	Malawi Institute of Education
MTPDS	Malawi Teacher Professional Development Support
MoESC	Ministry of Education, Sports and Culture
MoEST	Ministry of Education Science and Technology
MGCDSW	Ministry of Gender, Children, Disability and Social Welfare
MLA	Monitoring Learning Achievements
MSCE	Malawi School Certificate of Education
NCES	National Centre for Education Statistics
NCTM	National Council of Teachers of Mathematics
NI	Numeral Identification
NWA	Number Word After
NWB	Number Word Before
PSLCE	Primary School Leaving Certificate of Education

SACMEQ	Southern Africa Consortium for Monitoring Education Quality
SADC	Southern Africa Development Community
SEAL	Stages for Early Arithmetical Learning
TTC	Teachers' Training College
UEW	University of Education, Winneba
UNESCO	United Nations Education, Scientific, and Cultural Organization
UNICEF	United Nations for International Children Education Fund
USAID	United States Agency for International Development
ZPD	Zone of Proximal Development

CHAPTER 1

INTRODUCTION AND BACKGROUND TO THE STUDY

1.0 Chapter overview

This chapter presents the introduction and contextual background to the study by discussing the context of primary mathematics education in Malawi. This is followed by statement of the problem, purpose of the study, research questions, significance and limitations of the study. It concludes by presenting an overview of subsequent chapters of the thesis.

1.1 Introduction

Over the years, research into the area of number knowledge possessed by children in the early years of elementary school has increased (Wright, 1991; Payne & Huinker, 1993; Rumiati, 2010). Such research is important because effective teaching of mathematics requires an understanding of what children already know and need to know (National Council of Teachers of Mathematics [NCTM], 2000).

Most learners view mathematics as difficult, and that it has no meaning in their real life (Countryman, 1992; Van de Walle, 2001, Kazima & Adler, 2006). Learners begin to develop this perception from elementary school where they find the subject abstract and mostly relying on algorithms, which they fail to understand. Countryman (1992) argues

that the rules and procedures for school mathematics make little or no sense to many learners. Learners memorise examples, follow instructions, do their homework, and take tests, but cannot understand what their answers mean. Burns (1994) adds that imposing the standard algorithms on children gives the idea that mathematics is a collection of mysterious and often magical rules and procedures that require memorisation and practice. Consequently, low-attaining children develop strong negative attitudes towards mathematics due to lack of understanding and a rare experience of success in school mathematics (Wright, Martland & Stafford, 2000; 2006).

Every child arrives on the first day of school with lots of informal or intuitive mathematical knowledge (Smith, 1998; Kilpatrick, Swafford & Findell, 2001; Askew & Wiliam, 1995; Gervasoni, 2007). Children engage in all kinds of everyday activities that involve mathematics (Anderson, 1997), and consequently develop a wide range of informal knowledge (Perry & Dockett, 2004; Baroody & Wilkins, 1999). From infancy to preschool, children develop a base of skills, concepts and understandings about numbers and mathematics (Clarke & Clarke, 2006). Perry and Dockett (2002) noted that:

Much of this learning has been accomplished without the ‘assistance’ of formal lessons and with the interest and excitement of the children intact. This is a result that teachers would do well to emulate in our children’s school mathematics learning (p. 96).

Unfortunately, this so-called informal mathematical knowledge is often ignored by teachers when these children start school (Orton, 2004), and mathematical procedures have often been imposed on learners in ways that do not necessarily develop their

mathematical thinking or understanding (Carpenter, Ansell, Franke, Fennema & Weisbeck, 1993; Huinker, 1998). As a result, strategies for operating with numbers which teachers introduce may run counter to the knowledge brought to school by the children, and this may cause regrettable and unnecessary disequilibrium (Carpenter & Moser, 1982; MacNamara, 1990). Therefore, a significant proportion of children have difficulties learning basic arithmetic during early years of elementary school (Wright, Martland & Stafford, 2000).

From a traditional perspective, the teaching of mathematics has put much emphasis on teaching formal algorithms to children (Susuwele-Banda, 2005; Rumiati, 2010). With traditional approach to teaching, the majority of teaching tasks that have taken place in mathematics lessons have been based on the view that it is easy to transmit knowledge from the teacher to the learner, and that what is received is an exact copy of what was transmitted (Orton, 2004). This, however, is not the case. One major reason why children fail to achieve lasting learning is that the transmitted knowledge was never comprehensively grasped by the learner in the first place. Transmission learning often only achieves limited success, and the severity of the limitations may not be discovered until much later, or indeed may never be discovered at all (Orton, 2004).

From a constructivist learning perspective, children learn best if they are given opportunities to construct knowledge from their previous and current experiences (Cobb, 2000; Wright, 2004; Clements & Sarama, 2009). Vygotsky's constructivist theory of learning, related to his notion of the zone of proximal development (ZPD) (Vygotsky,

1978), proposes that children learn best if they are challenged within close proximity to, and slightly above, their current level of development. So, it is important to teach learners by meeting them where they are to achieve meaningful learning.

1.2 Background to the problem

All over the world, mathematics is widely understood to play a key role in shaping how people successfully deal with the various spheres of private, social, and civil life (Adler, Ball, Krainer, Lin & Jowotna, 2005; Anthony & Walshaw, 2009). The importance of mathematics skills continues to increase as societies and economies move towards more technologically advanced activities (Reubens, 2009). New learning goals in the field of mathematics are being advocated and also new recommendations for research are emerging (Fuson, 2004; US Department of Education/National Centre for Education Statistics [NCES], 2008). As people's knowledge increases through research and evaluation of programmes, they learn what works and what does not. In addition, they establish what children need to know as a foundation to become successful in learning mathematics in later years.

The skills and understanding needed for living in the 21st century are different from those expected of previous generations (Askew & Brown, 2001). Giving the right answer only in mathematics no longer serves as proof that a learner has understood a mathematical concept (Ghazali, Othman, Alias & Saleh, 2010). Rather than mastering a step-by-step procedure for written calculations, learners should invent their own mental strategies and make connections between real life problems and their corresponding mathematical

representations. The teacher's role, in this case, is to create a link between children's ability to use informal mathematics and the ability to understand the more formal mathematics found in school (Ginsberg, 1996). Teachers must help children construct and elaborate upon what they already know, so they can "re-invent" mathematics for themselves.

1.3 State of primary mathematics in Malawi

In Malawi, primary education consists of formal and informal primary schooling. The informal primary education constitutes largely pre-primary school which has an average age range of 3-5-year old children. Pre-primary schooling to the children of Malawi falls under the responsibility of the Ministry of Gender, Children, Disability and Social Welfare (MoEST, 2010), while primary schooling is under the Ministry of Education. The starting age of pre-primary schooling is 3 years and the duration is three years. At the moment, pre-primary schooling is provided by communities and private groups which own almost all of the Early Childhood Development (ECD) centres called Community Based Childcare Centres (CBCCs). According to MoEST (2008), only 30% of targeted CBCCs/pre-school children attend ECD centres.

The majority of these ECD centres are concentrated in urban and semi-urban areas. Most of the 4 to 5 year old children in rural communities do not have opportunities to have school readiness preparation due to absence of ECD centres in these areas (MoEST, 2010).

The formal primary school in Malawi has an eight-year cycle which runs from Standard 1 through Standard 8. This level is divided into three sections: infant section which comprises Standards 1 and 2; junior section which consists of Standards 3, 4 and 5; and senior section comprising Standards 6, 7 and 8 (Chimombo, Kunje, Chimuzu & Mchikoma, 2005). At the end of this eight-year cycle, learners sit for Primary School Leaving Certificate Examinations (PSLCE).

In 1994, after the first democratic elections in Malawi, the government of Malawi declared primary education free. This saw enrolment sharply rising from 1.2 million in 1994 to 3 million (Kunje, Selemani-Meke & Ogawa, 2009). By 1999 net enrolment had reached 99 percent (United Nations Statistics Division, 2010). The introduction of Free Primary Education (FPE) resulted in shortage of classrooms, teaching and learning resources and qualified teachers. It is, therefore, common in Malawi to see learners learning under a tree and with no textbooks (Susuwele-Banda, 2005). However, by 2004, improvements such as a decrease in the number of untrained teachers and provision of text books were made (Kunje, Selemani-Meke & Ogawa, 2009).

Studies have shown that Malawian children perform far below curriculum expectations. The 1999 Monitoring Learning Achievements (MLA) (Chinapah, 2000b; Ministry of Education, Sports and Culture [MoESC], 1999) survey conducted in eleven African countries including Malawi showed that Malawian learners, alongside those from the other countries, failed to reach an 80% numeracy target, which was set at the 1990 Jomtien world conference in Thailand. The desired mastery level for the survey was set at

73% performance score and the countries did not just fail to achieve the numeracy set target but were far below it (Susuwale-Banda, 2005). The mean score for Malawi in numeracy was 43%.

In 2002, the Southern Africa Consortium for Monitoring Education Quality (SACMEQ) II in 2002 carried out a study on Grade 6 learners in 15 countries including Malawi. The findings revealed that Grade 6 learners in Malawi scored 432.9 (SE = 2.25), which was far below 500 (SD = 100), the mean for all of the SACMEQ countries (Chimombo, Kunje, Chimuzu & Mchikoma, 2005). Malawian learners' achievement was the second lowest among the 15 SACMEQ countries (World Bank, 2010). In addition, the study found that 97.8% of the learners did not possess skills beyond basic numeracy level while 0% had skills beyond competent numeracy level. This meant that no learner in Malawi could conduct multiple steps with a range of basic operations because they were not mathematically skilled and lacked problem solving skills.

In 2008, MoEST conducted a national Primary Achievement Sample Survey (PASS) across all the 6 Education Divisions in the country (MoEST, 2010). The aim of the survey was to assess the context, conditions and conduct of public primary schools in relation to learner achievement levels particularly in mathematics and English in Standards 3, 5 and 7 (MoEST, 2010; Chimombo, Chiuye, Chide & Chiunda, 2014). The study involved 10% of the schools and used 10, 067 pupils as a sample. The results were that in mathematics less than 8% of the learners attained the grade level proficiency and competences expected in standard 3 and none of the learners scored above 50% in

standard 5. In standard 7 the results were not good as well as 99% scored below 50%. These studies consistently show that learners' underachievement in mathematics is a serious problem in Malawi.

The Malawi primary sector has other challenges as well. Dropout rates are high particularly in the first three classes of the primary subsector. According to the World Bank (2010), the percentage of children completing primary school in Malawi in 2010 was only 35%. This meant that 65% of the pupils were either repeating classes or dropping out of school. In addition, the average dropout rate (14.6%) and repetition rate (19.5%) were still high while survival rates for Standards 5 and 8 went down from 76.2% and 52.1% in 2008 to 64.5% and 31.5% in 2014 respectively (MoEST, 2014; World Bank, 2014). High dropout and repetition rates point to poor quality education, and in some cases poor teaching, poor infrastructure, unstimulating school environment and uneventful learning (MoEST, 2000; Chimombo et al., 2005; Chimombo et al., 2014). Indeed, a study by USAID (2014) on class repetition and attrition in Malawian primary schools revealed that 37.0% of the teachers in the sample indicated that teachers were also responsible for learners' class repetition due to poor quality teaching.

However, Malawi strives for quality education. In an attempt to improve the quality of education and increase the number of qualified teachers, the country has shifted from one teacher training programme to another (Susuwele-Banda, 2005; Khamisi & Kunje, 2011; MoEST, 2007). In addition, the country adopted an outcomes-based education (OBE) national primary curriculum both at primary school and teacher training college in 2007

as one way of improving the quality of primary education (MoEST, 2009; InWEnt, 2008). This change in curriculum resulted in a paradigm shift from teacher-centred methods to learner-centred methods of instruction in classrooms to make educational experiences more meaningful to learners and empower them to be active constructors of their own knowledge. In spite of all these initiatives, primary school learners still continue to achieve poorly in mathematics.

1.4 Statement of the problem

Malawi, like most developing countries, strives to improve the quality of its education. However, studies conducted in Malawi have collectively shown that primary school learners' achievement in mathematics is low and that many learners are failing to reach minimum levels of proficiency as specified in the Malawi primary national curriculum.

In general, these studies have investigated primary school learners' achievement in mathematics as well as factors affecting their performance. They have not clearly addressed the area of number knowledge possessed by children in Malawi. So, it is not clear how children in lower primary school in Malawi acquire number concepts. Considering the importance of mathematics and the learners' poor performance in mathematics in Malawi currently, there is need to gain a deeper understanding and insight into the children's learning of number in order to adequately support them as part of the country's quest in improving teaching and learning of primary school mathematics.

1.5 Purpose of the Study

The purpose of the study was to explore children's learning of number in lower classes in a Malawian primary school.

1.6 Main research question

The study was guided by the following main research question: How do children in lower classes in a Malawian primary school learn number?

1.6.1 Critical research questions

The following critical research questions were used to answer the main research question:

1. What strategies do mathematics teachers use to teach children early number concepts (counting, addition, and subtraction)?
2. How do learners understand number concepts (counting, addition and subtraction)?
3. What strategies do children use to count, add and subtract numbers?
4. What levels have learners reached in basic numeracy (numerical identification, forward and backward number word sequences, number word after and number word before)?

1.7 Significance of the study

In the area of children's learning of number in lower classes in Malawian primary schools, not much has been researched on. As such, little is known about children's acquisition of number concepts. So, findings from this study will give a picture about

how children acquire number concepts in lower classes in Malawian primary schools. This would in turn inform classroom practice on the teaching of number concepts in lower primary school in Malawi.

In addition, the findings of the study will benefit curriculum developers who would subsequently be able to design training programmes that take into account children's acquisition of number concepts in these classes in Malawian primary schools. Such programmes would help produce mathematics teachers who are adequately prepared to teach the concepts of number in lower classes in Malawian primary schools. Furthermore, most studies on children's acquisition of number knowledge have been done in developed countries. So, findings from this study will contribute to the growing body of literature that seeks solutions to the many numeracy and mathematics education challenges that children in most parts of the world continue to face.

1.8 Limitations of the study

One limitation was that the study used a case study approach within a qualitative research design with a small sample of 12 learners and 5 mathematics teachers. As such, the sample was not representative of all primary school learners and teachers in Malawi. So, the results would not be generalisable. However, the use of a qualitative research design meant that the aim of the study was to understand the phenomenon under study and not generalise its findings (Mukherji & Albon, 2010).

Nevertheless, if the study was conducted with a bigger sample and diverse geographical locations it would have given a better picture of children's acquisition of number knowledge in Malawian primary schools.

Another limitation was that the researcher's position as a teacher educator might have influenced research participants especially teachers to provide, during interviews, information they felt the researcher wanted to hear. The use of observations and documentary analysis helped overcome this problem.

1.9 Assumptions of the study

One assumption was made prior to conducting this study. It was assumed that both teachers' and learners' behaviours would not change because of the researcher's presence. While some influence is accepted as inevitable, it was considered that the influence would not invalidate the findings in relation to the focus of the study. Among other things, the researcher explained the aim of the study to all the research participants and they understood very well and took part willingly. In addition, data were triangulated.

1.10 Definition of operational terms

The following terms have been defined in order to allow for a clear understanding of the context in which they have been used in the thesis.

Number sense: A person's general understanding of numbers and operations along with the ability and inclination to use this understanding in flexible ways to make

mathematical judgments and to develop useful and efficient strategies for managing numerical situations (Reys & Yang, 1998; McIntosh et al., 1992).

Strategy: Broadly means any procedure a child uses to solve an arithmetic problem that can result in a correct answer. Children may count on fingers, try to retrieve the answer from memory, mentally calculate the answer to arithmetic problems, or use the algorithms taught in classrooms (Carr & Alexeev, 2011).

Mathematical proficiency: A term used to capture completely all aspects of expertise, competence, knowledge, and facility in mathematics, which make anyone to learn mathematics successfully (Kilpatrick, Swafford & Findell, 2001).

Algorithm: In number work, a standard, written procedure for doing calculation, which, if followed correctly, step by step, will always lead to the required result (Haylock & Manning, 2014).

Problem: In mathematics, a situation consisting of some givens and a goal, with a cognitive gap between them; this constitutes a problem for an individual, as opposed to just an exercise, if the way to fill the gap between the givens and the goal is not immediately obvious (Haylock & Manning, 2014).

1.11 Chapter summary

This chapter has discussed the background to the study in relation to the status of primary school mathematics in Malawi since the introduction of Free Primary Education in 1994. The chapter has also presented a statement of the problem, purpose of the study, research questions, significance and limitations of the study. In order to appreciate some of the studies that have been done in the area of children's number knowledge, the next chapter gives a review of relevant literature on the same.

1.12 Thesis structure

This thesis is presented in five chapters. Chapter one gives the introduction of the whole thesis. Chapter two reviews related literature and discusses the conceptual framework for the study. This is followed by Chapter three which gives an outline of the methodology of the study and its justification. Chapter four presents and gives a discussion of the research findings. Finally, Chapter five draws conclusions and gives implications and recommendations of the study.

CHAPTER 2

LITERATURE REVIEW

2.0 Chapter overview

This chapter discusses literature related to planning for teaching number sense, teaching for number sense, children's acquisition of number knowledge and constructivism. It also discusses literature on cognitive development theories as proposed by Piaget, Vygotsky and Ausubel. The chapter also discusses literature on children's understandings of basic arithmetic, children's thinking strategies, counting abilities, place value and base-ten structures. It also highlights literature on number, number words and numerals, and global perspectives on children's number knowledge. Finally, the chapter gives a conceptual framework that informed this study.

2.1 Planning for teaching number sense

Shulman (1987) conceptualized effective teaching as an amalgamation between content and pedagogy, while understanding how particular topics, problems or issues are organized and adapted to the diverse abilities of learners. Pedagogical content knowledge, for a particular topic concerns the teacher's knowledge of what makes the topic easy or difficult for learners to understand (Shulman, 1986). In addition, it includes knowledge of strategies that are most likely effective in reorganising learners'

understanding and eliminating their misconceptions, and a variety of effective ways of representing the ideas included in the topic such as analogies, illustrations, or examples. Therefore, to teach effectively, teachers need to be knowledgeable about not only number but also children's acquisition of those number concepts. In this way, they would be able to transform the content in ways that learners would easily understand. Teachers demonstrate all these in the way they draw schemes of work and prepare lesson plans.

How teachers organise classroom instruction is dependent on what they know and believe about mathematics and on what they understand about the teaching and learning of mathematics (Anthony & Walshaw, 2009). Teachers' sound content knowledge enables them to represent mathematics as a coherent and connected system (Ball & Bass, 2000). They use their knowledge to make key decisions concerning mathematical tasks, classroom resources, talk, and actions that feed into or arise out of the learning process. No matter how good their teaching intentions, teachers must work out how they can best help their learners acquire core mathematical ideas (Hill, Rowan & Bass, 2005). However, it is difficult for teachers to best assist learners to acquire core mathematical ideas like number concepts. They first need to know how children learn those concepts so that they can provide them with the necessary support. Therefore, it is necessary to carry out a study in the area of children's learning of number knowledge.

2.2 Teaching for number sense

Research on teachers' ideas about mathematics (Griffin & Case, 1997) reveals that many teachers regard mathematics as a fixed body of knowledge involving numbers and their

manipulation through rules and algorithms (Jackson, 1986). By treating numbers as disembodied entities, instruction focuses on ensuring that learners know various mathematics rules and the applications of those rules. Although numbers and algorithms are clearly involved in the business of doing mathematics, they are not the whole story. Mathematicians and enlightened educators view mathematics as a set of conceptual relationships between quantities and numerical symbols (Griffin, 2004), a view that underpins the ‘Principles and Standards for School Mathematics’ developed by the National Council of Teachers of Mathematics (2000).

Mathematics consists of three worlds: the actual quantities that exist in space and time; the counting numbers in the spoken language; and formal symbols, such as written numerals and operation signs (Griffin, 2004). Number sense requires the construction of a rich set of relationships among these worlds (Griffin, 2004). Children need, first, to link the real quantities with the counting numbers. Only then can they connect this integrated knowledge to the world of formal symbols and gain an understanding of their meaning.

Three instructional principles lie at the heart of teaching number sense and the number worlds programme (Griffin, 2004). Firstly, is providing rich activities for making connections. Rich activities expose children to the three worlds of mathematics — quantities, counting, and formal symbols— and to multiple opportunities for constructing relationships among the three worlds. They also introduce children to five different forms of number representation namely, groups of objects, dot-set patterns and numerals, position on a path or line, position on a vertical scale, and position on a dial. Secondly, is

giving children opportunities to actively explore and discuss numerical concepts in a social context such as in groups. Working actively in collaborative settings provides many opportunities for children such as argumentation and reasoning. Finally, is ensuring an appropriate sequence of concepts. A carefully graded sequence of activities enables children to use their current understandings to construct new ones. A seamless sequence of activities permits individual children to start at an appropriate individual level and to move through the normal developmental progression at a suitable pace. In this way, children encounter many opportunities to use numbers to make sense of quantity representations.

Muir (2008) developed and refined an observation schedule designed to evaluate effective teaching for numeracy. She categorised teachers' efforts such as making connections, choice of examples, use of concrete materials, mental computation and the use of open-ended questions as important components in effective teaching for numeracy. Rich instructional environments have been linked to instruction that builds on children's ways of thinking mathematically (Clarke, 2008; Sowder, 2007) and documented gains in children's achievement (Bobis, Clarke, Clarke, Thomas, Wright & Gould, 2005). As such, studying children's acquisition of number concepts is more informative.

Some studies have specifically looked at the teaching of mathematics through cognitively guided instruction (CGI). CGI is a professional development programme which focuses on the development of children's mathematical thinking; instruction that influences that development; teachers' knowledge and beliefs that influence their instructional practices;

and the way that teachers' knowledge, beliefs, and practices are influenced by their understanding of children's mathematical thinking (Carpenter, Fennema, Franke, Levi & Empson, 1999). In addition, CGI engages teachers in learning about the development of children's mathematical thinking within particular content domains.

In their first CGI experimental study of teachers' knowledge and beliefs about children's thinking, Carpenter, Fennema, Peterson and Carey (1988) found that teachers' knowledge of their children's thinking was related to learner achievement. Children of teachers who knew more about their children's thinking had higher levels of achievement in problem solving than children of teachers who had less knowledge of their children's thinking. In a related study, Peterson, Fennema, Carpenter and Loef (1989) found that classes of teachers whose beliefs were more consistent with principles of CGI tended to have higher levels of children achievement than classes of teachers whose beliefs were less consistent with principles of CGI. CGI teachers placed greater emphasis on problem solving and less on computational skills, expected more multiple-solution strategies rather than a single method, listened to their children more, and knew more about their children's thinking than did control teachers.

Whereas the initial experimental study compared different groups of teachers, a three year longitudinal study of 21 teachers (Fennema, Carpenter, Franke, Levi, Jacobs, Empson, 1996) explicitly examined the nature and pattern of change among teachers and the relationship between beliefs and instruction. Several levels of beliefs and practice in becoming a CGI teacher were identified. Teachers operating at Level 1 believe that

children need to be explicitly taught how to do mathematics. Instruction in their classes is usually guided by an adopted text and focuses on the learning of specific skills. Teachers generally demonstrate the steps in a procedure as clearly as they can, and the children practice applying the procedures. Children are expected to solve problems using standard procedures, and there is little or no discussion of alternative solutions. Level 2 teachers begin to question whether children need explicit instruction in order to solve problems, and the teachers alternately provide opportunities for children to solve problems using their own strategies and show the children specific methods.

Level 3 is a turning point. At this level, teachers believe that children can solve problems without having a strategy provided for them, and they act accordingly. Children spend most of mathematics class solving and reporting their solutions to a variety of problems. Classrooms are characterised by children talking about mathematics, both to other children and to the teacher. They report a variety of strategies and compare and contrast different strategies. Mathematics classrooms are strongly influenced by teachers' understanding of children's thinking, they know appropriate problems to pose and questions to ask to elicit children's thinking, and they understand and appreciate the variety of solutions that children construct to solve them.

Carpenter et al (1999)'s CGI has three basic tenets: instruction that is based on the learners' current knowledge, instruction that is based on children's understanding of mathematics, and instruction that takes into account the mentally active mind-set of the learner. These studies show that developing an understanding of children's thinking

provides a basis for change, but change occurs as teachers attempt to apply their knowledge to understand their own learners. In addition, the studies show not only how teachers can change by learning about children's thinking but also demonstrate how much can be accomplished by both teachers and learners when learners' thinking becomes a primary focus of instruction (Carpenter et al., 1989). So, a study into children's learning of number concepts is important as it would reveal not only how learners understand number concepts but also strategies they use to learn such concepts. This would help teachers build on the strategies learners already know for effective teaching of number concepts.

Askew, Brown, Rhodes, Johnson and Wiliam (1997) classified characteristics of effective teachers into four categories. The first category is about organisational and management strategies. These include how time on task is maximised, catering for collective and individual needs, and coping with range of attainment. The second category is about teaching styles. They refer to intervention strategies, questioning techniques, quality of explanations, assessment of attainment and understanding as well as handling children's errors. The third category concerns teaching resources, which include sources of activities, range of tasks, availability of resources and expected outcomes. The fourth and last category is about pupil responses, which include ways of working and evidence of understanding. Teachers' understanding of children's mathematical thinking and understanding of number concepts requires not only paying attention to children's strategies but also interpretation of the mathematical understandings reflected in those strategies.

All these studies suggest that teaching for number sense requires specific knowledge for teachers to make instructional decisions that benefit their learners. Therefore, it is important to study how children learn number.

2.3 Constructivism

Constructivism is a theoretical perspective that proposes that children “actively construct (rather than passively absorb) knowledge from their experiences” (Ormrod, 2014, p. 27). This knowledge, once constructed, is organised into schemes, which are “groups of similar actions or thoughts that are used repeatedly in response to the environment” (Ormrod, 2014, p. 27).

There are two main sources of knowledge construction, namely psychoanalytical (Kelly, 1955) and Piaget’s cognitive constructivism. Kelly (1955) emphasises on the personal construction of knowledge and argues that “a person’s processes are psychologically channelized by the way in which they anticipate events” (Barnister & Fransella, 1986, p. 7). He asserts that people have theories about the world which form the basis from which they seek to predict what will happen next. They seek to understand their own nature and the nature of the world and to test that understanding in terms of how it guides and enables them to see the immediate and long term future (Barnister & Fransella, 1986). Just like what a scientist does, human beings are said to be innately curious, and seek to make sense of the world and their own existence and operate their daily lives.

Piaget, on the other hand, emphasises on the construction of reasoning patterns, and describes the process of knowledge construction in terms of adaptation. Piaget's theory is described below.

2.3.1 Piaget's theory of cognitive development

Jean Piaget (1896-1980), a Swiss psychologist, devised a model that describes how humans go about making sense of their world by gathering and organising information (Piaget, 1963). As a result of his early research in biology, Piaget concluded that all species inherit two basic tendencies or invariant functions, namely organisation and adaptation. Organisation involves “combining, arranging, recombining and rearranging of behaviours into coherent systems” (Woolfolk, 2007, p. 28). On the other hand, adaptation is concerned with “adjusting to the environment” (Woolfolk, 2007, p. 28).

2.3.1.1 Organisation

People are born with a tendency to organise their thinking processes into psychological structures called schemes, which are “the basic building blocks of thinking” (Woolfolk, 2007, p. 28). They allow human beings to mentally represent or think about the objects and events in their world.

2.3.1.2 Adaptation

Piaget saw the process of cognitive development as taking place through the process of adaptation. Adaptation involves two basic complementary processes: assimilation and accommodation. Assimilation takes place when people use their existing schemes to

make sense of their world (Woolfolk, 2007). In this regard, new information is fitted into what they already know. Accommodation “is a process of dealing with a new object or event by modifying an existing scheme or forming a new one” (Ormrod, 2014, p.27). This happens when a child cannot easily interpret or respond to a new object or event using existing schemes. The process of accommodation then leads to restructuring of the child’s existing schemes which leads to development, as the child is now able to cope with more complex ideas.

2.3.1.3 Equilibration

According to Piaget, organising, assimilating and accommodating form a kind of complex balancing act. The actual changes in thinking are said to occur through the process of equilibration, which is “the act of searching for a balance” (Woolfolk, 2007, p. 29). Piaget proposed that children are in a state of equilibrium, “state of being able to to address new events with existing schemes” (Ormrod, 2014, p. 27). However, as they grow older and expand their horizons, they sometimes meet situations which they cannot deal with their existing schemes. Such situations create a state of disequilibrium, a sort of mental discomfort that spurs them to try to make sense of what they are observing (Ormrod, 2014). Provided that the level of disequilibrium is optimal, children are motivated to keep searching for a solution through assimilation and accommodation, and thus their thinking changes and moves ahead.

2.3.2 The stage theory of cognitive development

Piaget proposed that as a result of brain maturation, innumerable experiences, and children's natural desire to make sense of and adapt to their world, cognitive development undergoes qualitative changes, which proceed in four distinct stages (Piaget, 1971). The four stages are hierarchical, each one providing a foundation for any subsequent stages (Berk, 1997; Ormrod, 2014). All people pass through each stage before starting the next one and the thinking characteristics of each stage determine what a child can learn from their experiences. Piaget named these stages of development broadly as sensorimotor, preoperational, concrete operational and formal operational stages. Although children are usually grouped by chronological age, their development levels may differ significantly (Weinert & Helmke, 1998), in addition to the rate at which individual children pass through each stage (Ojose, 2008). This difference may depend on maturity, experience, culture, and the ability of the child (Papila & Olds, 1996).

2.3.2.1 Sensori-motor stage (from birth to 2 years)

During this stage, the child moves from reflex actions to goal directed activity (Woolfolk, 2007). In addition, the child begins to use imitation, memory and thought. Further to that, the child begins to recognise that objects continue to exist even when hidden (object permanence). He or she also begins to learn to link numbers to objects (Piaget, 1977), such as one dog, three pigs (Ojose, 2008). Studies suggest that during this stage, children have some understanding of the concepts of numbers and counting (Fuson, 1988).

So, educators of children at this stage should lay a solid mathematical foundation by providing activities that incorporate counting activities to enhance children's conceptual development of number (Ojose, 2008).

2.3.2.2. Pre-operational stage (from 2 to 7 years)

During this stage, the child gradually develops use of language and ability to think in symbolic form. His or her thinking is prelogical, and rational thought makes little appearance. The child does not understand point of view (Ormrod, 2014), links together unrelated events, sees objects as possessing life, and cannot reverse operations (Ojose, 2008; Wadsworth, 1996). Children's perceptions in this stage are generally restricted to one aspect or dimension of an object at the expense of the other aspects. As such, the child cannot conserve number, mass, length or volume: he or she cannot solve transitive, class inclusion, or seriation problems. Seriation is the ability to order objects according to their length, weight, or volume whereas classification involves grouping objects on the basis of their common characteristics (Ojose, 2008). So, teaching and learning experiences in this stage of development should employ effective questioning about characterizing objects (Ojose, 2008). Engaging children in discussion or interactions may engender them to discover a variety of ways to group objects, thus helping them think about the quantities in novel ways (Thompson, 1990).

2.3.2.3 Concrete operational stage (from 7 to 11 years)

During this stage, the child now understands reversibility, laws of conservation and is able to classify and seriate (Piaget, 1977) and both are essential for understanding number concepts. The child is also able to solve problems logically but still needs concrete

objects for support. The need for concrete objects still implies that providing learners with “hands-on experiences and multiple ways of representing a mathematical solution can be ways of fostering the development of this cognitive stage” (Burns & Silbey, 2000, p. 55). Hands-on activities provide children an avenue to make abstract ideas concrete, allowing them to get their hands on mathematical ideas and concepts as useful tools for solving problems (Ojose, 2008; Baroody, 1987).

As children use the materials, they acquire experiences such as seeking relationships, making patterns, counting, and sorting that help lay the foundation for more advanced mathematical thinking by promoting reflective thinking in young children (Payne, 1990). Furthermore, children’s use of materials helps to build their mathematical confidence by giving them a way to test and confirm their reasoning. However, children may not automatically make connections between the work they do with manipulative materials and the corresponding abstract mathematics. They “tend to think that the manipulations they do with models are one method for finding a solution and pencil-and-paper math is entirely separate” (Burns & Silbey, 2000, p. 60). Therefore, teachers need to bridge these activities in order to help learners understand mathematical concepts.

2.3.2.4 Formal operational stage (from 11 years to adulthood)

A child in this stage is able to solve abstract problems in logical fashion. He or she is capable of forming hypotheses and deducing possible consequences, allowing the child to construct his or her own mathematics (Ojose, 2008). In addition, the child typically begins to develop abstract thought patterns where reasoning is executed using pure

symbols without the necessity of perceptive data. The sort of reasoning skills refer to the mental process involved in the generalising and evaluating of logical arguments (Anderson, 1990) and include clarification, inference, evaluation, and application.

Clarification requires children to identify and analyse elements of a problem, allowing them to decipher the information needed in solving a problem. By encouraging children to extract relevant information from a problem statement, teachers can help children enhance their mathematical understanding. Children at this stage are developmentally ready to make inductive and deductive inferences in mathematics. Deductive inferences involve reasoning from general concepts to specific instances. On the other hand, inductive inferences are based on extracting similarities and differences among specific objects and events and arriving at generalisations. Evaluation involves using criteria to judge the adequacy of a problem solution. For example, the child can follow a predetermined rubric to judge the correctness of his solution to a problem. Evaluation leads to formulating hypotheses about future events, assuming one's problem solving is correct thus far. Finally, application involves children connecting mathematical concepts to real-life situations. It is important, however, to bear in mind that many children are apt to be in transition from one stage to the next, displaying characteristics of two adjacent stages at the same time (Ormrod, 2014).

2.3.2.5 Implications of Piaget's theory

Piaget's work has received a number of criticisms such as underestimating the abilities of young children (Gelman, Meck & Merkin, 1986), failing to offer a complete description of cognitive development (Eggen & Kauchak, 2000), and proposing that cognitive development cannot be accelerated (Zimmerman & Whitehurst, 1979; Adey & Shayer, 1990). Piaget has also been criticized for overestimating the abilities of older learners, having implications for both learners and teachers. For example, middle school teachers interpreting Piaget's work may assume that their students can always logically in the abstract, yet this is often not the case (Eggen & Kauchak, 2000). Despite heavily criticised, the theory has been most influential in its contribution to general teaching and teaching to enhance thinking (Adey & Shayer, 1993; Lawson, 1985). Outlined below are most notable of his contributions:

2.3.2.5.1 Children as active learners

The idea that children actively construct their understanding from interaction with the physical world has contributed much to teaching. It has led to a shift from teacher centred methods to child centred and discovery methods. As such, instruction has been characterised by lots of hands-on and experiences (Burns & Silbey, 2000) which make abstract ideas concrete and accessible to learners (Ojose, 2008).

The idea of assimilation and accommodation has highlighted the need for meeting learners where they are, and helping them build on what they know (Clements & Sarama, 2009). This is achieved by incorporating cognitive conflict to extend children's

understanding. In general, the knowledge of Piaget's stages helps the teacher understand the cognitive development of the child as the teacher plans stage-appropriate activities to keep children active (Ojose, 2008).

2.3.3 Vygotsky's theory of cognitive development

Lev Semenovich Vygotsky was a Russian scientist born in 1896 and died prematurely of Tuberculosis at the age of 38. His theories became available to the world outside Russia when they were translated from Russian to English in the 1960s. He was a contemporary of Piaget and shared his view of a child constructing meaning from interacting with the environment. He also proposed that a child's thinking was limited because of certain higher intellectual functions such as awareness of mental operations that are not available until adolescence. Vygotsky, however, differed from Piaget in that he gave much more emphasis to the role of social interaction and language in cognitive development. Vygotsky's writings have three main themes that explain how social processes form learning and thinking (Driscoll, 2005; Wertsch & Tulviste, 1992). These are the general genetic role of cultural development, the zone of proximal development, and the role of cultural tools in learning and development, especially language.

2.3.3.1 The general genetic role of cultural development

Vygotsky (1981) believed that higher-order mental processes, such as reasoning and problem solving, are first co-constructed during shared activities between the child and another person. Then the processes are internalised and become part of the child's cognitive development. Vygotsky (1978) argued that "every function in a child's cultural

development appears twice: first, on the social level and later on the individual level; first between people (interpsychological) and then inside the child (intrapsychological)” (p. 57). This underscores the role of social interaction in cognitive development. These social interactions are more than simple influences on cognitive development –they actually create the child’s cognitive structures and thinking processes (Palinscar, 1998). Development is seen as the acquisition of psychological tools that have been shaped by society’s historical development (Hedegaard, 1996). In this regard, Vygotsky conceptualised development as “the transformation of socially shared activities into internalised processes” (John-Steiner & Mahn, 1996, p. 192).

2.3.3.2 The zone of proximal development (ZPD)

Vygotsky believed that, at any point in development, there are certain problems that a child is on the verge of being able to solve. This child needs some structure, clues, reminders, help with remembering details or steps, encouragement to keep trying, and many more (Vygotsky, 1978). The zone of proximal development (ZPD) is the distance between a child’s “actual development level as determined by independent problem solving and the higher level of potential development as determined through problem solving under adult guidance or in collaboration with more able peers” (p. 86).

The idea of the ZPD is built upon the notion of mental functions arising from social interaction and the idea of cognitive development, which assesses children’s intellectual processes and evaluating instructional practices. The ZPD operates at three levels (Daniels, 1996). The first level concerns scaffolding, a term invented by Bruner, which

refers to the difference between the child's initial independent performance and final performance with assistance. According to Bruner, much teaching involves scaffolding, in which teachers support children in learning concepts and language.

The second level is cultural ZPD which relates to the development of concepts. Vygotsky distinguished scientific concepts from spontaneous or everyday concepts. Scientific concepts are seen to be highly organised and hierarchical whereas everyday concepts are seen to be tightly linked to particular contexts and lacking in overall system (Wertsch, 1985). Scientific concepts are said to be learned in a school setting, part of the cultural knowledge system, with explicit verbal definitions and where learning is consciously made. Spontaneous concepts, on the other hand, are said to be learned in the course of everyday life, where they are not consciously made, and are used with ease, without being aware that there is such a thing as a concept (Newman & Holzman, 1993). Therefore, learning is viewed as leading to scientific concepts whereas development is seen to lead to spontaneous concepts. The merging of scientific concepts and everyday concepts lead to mature concepts (Daniels, 1996), in which everyday concepts are seen to bring the richness and detailed patterns of everyday thinking into the system and organised structure.

The third level is social ZPD, which is seen as the distance between cultural knowledge provided by social historical context and everyday experience. It is understood and active knowledge.

2.3.3.3 The role of learning and development

Piaget defined development as the active construction of knowledge and learning as the passive formation of associations (Siegler, 2000). He was interested in knowledge construction and believed that cognitive development had to come before learning so that the child is cognitively ready to learn. He argued that “learning is subordinated to development and not vice versa” (Piaget, 1964, p. 17). In contrast, Vygotsky believed that learning is an active process that does not have to wait for readiness. In fact, Vygotsky argued that “properly organised learning results in mental development and sets in motion a variety of processes that would be impossible apart from learning” (Vygotsky, 1978, p. 90). He saw learning as a tool in development in which learning pulls development up to higher levels and social interaction is a key in learning (Glassman, 2001; Wink & Putney, 2002). Vygotsky’s belief that learning pulls development to higher levels means that other people, including teachers, play a significant role in cognitive development (Woolfolk, 2007).

2.3.3.4 The role of language and private speech

Vygotsky talked about thinking as a form of inner speech, which enables humans to plan and regulate their own action (Vygotsky, 1962). Vygotsky had very different ideas about young children’s private speech. Rather than being a sign of cognitive immaturity (as proposed by Piaget), he suggested that these mutterings play an important role in cognitive development by moving children towards self regulation, which is the ability to plan, monitor, and guide one’s own thinking and problem solving. Private speech is a transition from external to inner speech. Wertsch (1979) outlines four levels in which a

child makes a transition to inner speech. At Level 1, the child fails to interpret adult utterances referring to the task situation. At Level 2, the child responds to specific question, instruction or command in relation to the task. At Level 3, he or she is able to follow non explicit directives such as hints. Finally, at Level 4, the child can carry out the task alone and the verbal activities shift from the inter-psychological to the intra-psychological plane.

The first three levels occur within the ZPD and are viewed as a process of transfer of executive control from the adult to the child. The regulation of behaviour first develops in the context of adult-child interaction, in a shared act, and then the caregiver sensitively and gradually withdraws from the joint activity, allowing promotion and rewarding the child's take-over of the regulatory role (Diaz, Neal & Amaya-Williams, 1990).

2.3.4 Educational implications of Vygotsky's theory of cognitive development

The idea of the zone of proximal development (ZPD) in education has offered potential for cognitive acceleration and a way of structuring teaching and learning such that it is challenging to learners while at the same time not too difficult to cause frustrations. In this case, good instruction should take place where learning occurs within the ZPD, going ahead of development. The teacher is expected to facilitate during the teaching and learning process and model behaviour, such as self regulation and provide psychological tools, for asking questions, providing prompts and giving feedback. By doing this, the teacher helps children discover and communicate ideas that would not have occurred spontaneously without adult's help (Vygotsky, 1978).

2.3.5 David Ausubel's cognitive development theory

Ausubel proposed a cognitive development theory that deals with process such as concept formation and assimilation (Ausubel, Novak & Hanesian, 1978). Human beings “interpret raw perceptual experience in terms of particular concepts in their cognitive structures” (Ausubel, Novak & Hanesian, 1978, p. 86). In this case, Ausubel proposed that meaningful learning in human beings occurs through an interaction of new information with relevant existing ideas in the cognitive structure. This led to the proposal that “the most important single factor influencing in learning is what the learner already knows. Ascertain this and teach him or her accordingly” (Ausubel, 1968, p. 337). Ausubel made a distinction between two processes in learning. These are concept formation and concept assimilation. Concepts themselves “consist of the abstracted criterial attributes that are common to a category of objects, events or phenomena...” (Ausubel, Novak & Hanesian, 1978, p. 86). Concept formation involves inductive and spontaneous (untutored) acquisition of generic ideas from concrete-empirical experience. This is said to be the primary mode for concept acquisition in preschool and early elementary school years. On the other hand, concept assimilation involves learning new conceptual meanings by being presented with the criterial attributes of concepts and by relating these attributes to relevant established ideas in the cognitive structure. Concept assimilation is associated with meaningful reception learning, where the content of what is to be taught is presented to the learner in the final form and the learner is required to internalise or incorporate the material in her or his cognitive structure. Most of classroom learning is organised along the lines of reception learning and meaningful reception learning is the main means of acquiring large bodies of knowledge.

Meaningful reception learning takes place if the learning task can be related in a non arbitrary substantive fashion to what the learner already knows, and if the learner adopts a correspondingly learning set to do so. On the other hand, rote learning occurs if the learning task consists of arbitrary association and if the learner lacks the relevant prior knowledge for making the task potentially meaningful, also (regardless of how much potential meaning the task has) if a learner adopts a set merely to internalise in an arbitrary verbatim fashion (Ausubel et al., 1978, p. 27).

In developmental terms, Ausubel proposed that in preschool and early elementary school years, concepts are acquired mainly through concept formation and later elementary school years concrete because empirical props are necessary for concept assimilation. In the junior high school years, the learner can dispense with the concrete –empirical props and directly relate the presented criterial attributes to his or her cognitive structure. In addition, with increasing age, concepts tend to consist of more higher-order abstractions, more precision and differentiation. They tend to be acquired more through concept assimilation than through concept formation. Concept assimilation is accompanied by an awareness of the conceptualisation of the operations involved. Among others, Ausubel incorporates ideas of constructivism from Piaget, the idea of intellectual development, and the importance of language from Vygotsky. He proposed a theory of learning rather than a theory of cognitive development.

2.4 Children's acquisition of number knowledge

Over decades there has been a gradual change to mathematics instruction. The change has been strongly influenced by cognitive development theories as proposed by Jean Piaget and Lev Vygotsky among others (Cobb, 2000; Steffe & Thompson, 2000). Piaget's theory about constructivism has inspired many researchers to give more attention to

children's thought processes. For example, Von Glasersfeld, who developed radical constructivism as a theory of knowing, attributes Piaget with the founding of the epistemological theory of constructivism (Steffe & Thompson, 2000).

The constructivist learning perspective has impacted significantly in several countries and has challenged the traditional approach to teaching and learning. In contrast to the traditional approach to teaching and learning mathematics which regards a teacher as the sole source of children's knowledge, constructivists view children as the most important factor in their learning. This view assumes that children actively construct their knowledge and understandings from their interactions with the physical and social world (Vygotsky, 1981). In this case, knowledge is perceived as something that is constructed through interpretation and organization of information and not something that is merely transferred from a more knowledgeable source to a learner (Adams, 2007).

Influenced by Piaget's cognitive developmental theory especially in the area of children's conceptions of number, researchers such as Steffe and Cobb (1983) and Cobb and Wheatley (1988) investigated how children develop arithmetic strategies in the early years of school. In the early 1990s, several researchers, drawing on the earlier work by Steffe and Cobb, investigated the numerical knowledge and strategies of children beginning school and subsequent development of these aspects (Wright, 1991a; 1994; Askew & Wiliam, 1995). Common to these studies were that children begin formal schooling with intuitive or informal number knowledge and ways they use to count, add and subtract small numbers. In addition, the studies found that the number knowledge

and strategies possessed by children in the early years of school varied widely. For example, in Australia, a study by Wright (1994) described a three year difference in children's early number knowledge, that is, some 4-year-old children had attained a level of number knowledge and numerical strategies that others will not attain until they reach 7 years. This three-year difference shows that there are variations in terms of number knowledge possessed by children in elementary school classes. So, mathematics teachers need to be aware of these differences in order to design appropriate teaching material and that suits learners of mixed abilities.

Further to that, the studies revealed that, by and large, and in the absence of intervention, the children's differences in number knowledge increase as children progress through the early learning years of school and beyond (Aunola, Leskinen & Lerkkanen, 2004; Wright Martland & Stafford, 2000). A three-year difference in the early years of school becomes a seven-year difference for low attaining children after about ten years of school (Cockcroft, 1982; Wright Martland & Stafford, 2006). Therefore, these studies indicate how crucial the early years of school are in forming the foundation of children's mathematical learning. Unfortunately, the school system and mathematics teachers underestimate or overestimate children's numerical knowledge and strategies and their ability to manage their own learning (Orton, 2004). Therefore, to advance children's number knowledge and strategies to more sophisticated levels, assessments should be conducted to inform the teaching and learning. This is because teaching children number sense is most likely to be successful if it takes into account what they already (Wright, 2008; Clements & Sarama, 2009).

The emphasis on detailed assessment of children's numerical knowledge and strategies as a basis for teaching them well has strongly influenced recent initiatives in early number learning in several school systems, especially in developed countries. In Australia, based on the pioneering work of Wright (1989, 1991a, 1992), several systemic initiatives have been implemented to change the approach in teaching and learning number in the early years of primary school (Perry & Dockett, 2007). These include the Count Me in Too initiative in New South Wales, the Early Numeracy Programme in Victoria and the New Zealand Numeracy Development Project (Bobis, Clarke, Clarke, Thomas & Wright, 2005). The common feature to all these initiatives is the use of ongoing assessment to inform practice. Thus, assessment informs teaching and teaching provides additional assessment information. Wright, Martland and Stafford (2006, p. 6) give an outline of the principles involved in these initiatives based on the principles of the Mathematics Recovery Programme (MRP). These include the use of a guiding framework (Learning Framework in Number), the approach to assessment and the assessment tasks, the underlying theory of early numerical learning, the guiding principles for teaching, and approaches to teacher professional development.

2.5 Children's understandings of arithmetic-conceptual basis for informal arithmetic

2.5.1 Addition and subtraction

Children start formal schooling with some very basic addition and subtraction concepts (Reubens, 2009). Their fundamental understanding of addition and subtraction evolves from their early counting experiences (Gelman & Gallistel, 1978; Ginsburg, 1977). As

they play with collections of objects such as of one, two and three, children recognise that adding something to a collection makes it larger and taking away something from the collection makes it smaller. This knowledge is seen in children as young as 3 to 5 years (Cooper, Starkey, Blevins, Goth & Leitner, 1978; Starkey & Cooper, 1980).

In their study with older preschoolers, Sarnecka and Gelman (2005)'s findings continue this pattern of evidence, favouring the position that children know that the operations of adding and subtracting systematically increase and decrease the value of cardinality, even if they cannot reliably count the number of items involved. So too did Hartnett and Gelman (1998)'s study on understanding of the successor principle of 5- to 8-year-olds. These children were more likely to say that adding 1—as opposed to counting further—could go on indefinitely. Why? Although they knew that adding would increase the number, they did not know the words for those numbers, as illustrated by explanations two children gave: “You just can’t put it in the newspaper like, we thought of some new numbers”; and “There aren’t real numbers [meaning known terms] but you could make them up.”

From their numerous experiences that involve adding something to an existing collection to make it larger or removing items from a collection to make it smaller, children construct an informal conceptual basis for understanding addition as an incrementing process and subtraction as a decrementing process (Baroody & Wilkins, 1999). Preschoolers can use their incrementing (add-to) view of addition and decrementing (take-away) view of subtraction to comprehend and solve simple arithmetic tasks or word

problems (Gelman & Gallistel, 1978). Therefore, given the understanding of counting, children can notice that the repeated placement of one item into a collection increases its value (Gelman, 2006).

Studies conducted over decades have investigated children's knowledge of addition, and have focused on children's ability to solve addition problems. Some of the abilities assessed include the time it took to solve a problem, the size of the problem that was solved, and the strategy used in solving the problem (Groen & Parkman, 1972; Groen & Resnick, 1977). These studies confirm that children use a variety of strategies to solve problems like counting from one on fingers and counting from the larger addend (Siegler & Robinson, 1982). The studies also show children's ability prior to any schooling. For example, preschool children in the United States demonstrated the knowledge that the number that answers an addition problem is greater than the largest addend in the problem (Siegler & Shrager, 1984).

2.6 Children's thinking strategies

Research into children's thinking strategies for the basic number facts dates back to several decades ago (Thornton, 1978) and continues even today (Gervasoni, 2007; Wright, 2013). During the last three decades, the teaching and learning of mathematics has predominantly shifted from being one of 'transferring knowledge' to one of 'constructing knowledge' (Cobb, 2000). Along with this shift, the importance of counting in the development of children's strategies for solving number problems has been given much emphasis (Wright, 1991a).

Children's strategies in addition and subtraction problems have been studied by many other researchers (Fuson & Kwon, 1992b; Carpenter, Fennema, Franke, Levi & Empson, 1998; Baroody, 2007). Generally, these researchers agree that children's counting strategies can be classified into counting-all, counting-on from the first addend, counting-on from the larger, counting-back, and non-counting-by-ones. A child using count-all strategy represents each number in a mathematical problem with concrete objects such as counters fingers or slashes. Then, the child puts the sets of objects together and counts them from 1. A child using a count-on or count-back strategy is able to hold one number in mind and count on or back from that number while keeping track of the quantity that is added or subtracted using fingers, tally marks, or counters. A child using non-counting-by-ones strategies is able to solve problems in flexible ways, often breaking numbers down and recombining them by using known number facts. The child is able to do this because he or she understands number relationships.

These studies show that children use a variety of strategies to deal with number problems. However, the contexts in which these studies have been conducted are different from that of Malawi. In spite of this, it is important to carry out this study in Malawian primary schools to see how learners in a Malawian context learn number concepts.

2.7 Children's counting ability

Many children start school already able to count (Cotton, 2013). However, Cotton argues that it is important to be aware of the principles of counting, both to support the children who cannot yet count and to recognise the processes that young children who have already learnt to count have mastered.

Key research on how children learn to count was done by Gelman and colleagues such as Gelman and Gallistel (1978) and Gelman, Meek and Merkin (1986). They proposed that young children possess an innate concept of number consisting of a set of counting principles that define correct counting. Through careful observation of young children undertaking activities that they had planned, Gelman and Gallistel (1986) described five principles which underpin children's process of learning how to count. These principles are called Counting Principles Theory (Wynne, 1992).

The first one is one-to-one principle. A child who understands this principle knows that each item is counted only once with members of the set of number tags that are used to count with, such as a set of number words.

The second one is stable-order principle. In this principle, a child knows that the order of number names or number tags must have a fixed order in which they are consistently used. In this case, counting is always done "by saying one, two, three, four, five... in that order" (Cotton, 2013, p. 50). The third principle is cardinality principle, in which the child knows that the number they attach to the last object they count represents the

cardinality of the items counted. The fourth one is abstraction principle. With this principle, a child knows that they can count anything-they do not all need to be the same type of object.

Lastly is the order-irrelevance principle, in which a child knows that they can count a group of objects in any order and in any arrangement and they will still get the same number of items in a particular collection.

The counting principles theory asserts that children will possess the first item of knowledge at a very early age as soon as they have made the connection between the list of number words and their own list of mental number tags (Wynne, 1992). It also predicts that children will have the second item of knowledge for all and only the number words within their counting range. For instance, a child could not be expected to know which numerosity the word “three” refers to, if they do not know where in the number word list the word “three” falls. But a child who can count up to three, and knows that the words refer to numerosity, must know the specific numerosity that the word “three” picks out.

2.7.1 Uses of counting

The term “counting” is used in a particular sense. Cobb and Steffe (1983) contend that, with counting, a child coordinates the production of a sequence of number words with the production of a sequence of unit items (items that are equivalent for the child in some way). In this case, counting follows the establishment of a collection of countable items,

and results in a collection of counted items. Wright (2013) and Wright, Martland and Stafford (2000) add that a child achieves this by using a number word sequence (forward number word sequence or backward number word sequence). In this regard, "counting puts abstract number and simple arithmetic within the reach of the child" (Baroody, 1987, p. 33).

Steffe and colleagues (Steffe, Cobb & Von Glasersfeld, 1988; Steffe, 1992), proposed a model of construction of arithmetic strategies. Using tasks involving covered and uncovered collections of counters to present number problems to children, Steffe and colleagues constructed a theory of children's counting types. The theory divides children's counting into five types, namely; perceptual, figural, motor, verbal and abstract unit items. A perceptual counter is able to count visible items. A figural counter is a child who is able to count covered items by visualising images of those items. A motor counter produces movements as substitutes for visual items screened from view (that is substitute for the perceptual or figural items), and a verbal counter says the number words as a substitute for the countable items.

2.7.2 Combining and partitioning numbers

Counting strategies are an important aspect of children's early arithmetical knowledge. At the same time as they develop counting strategies, children may also develop knowledge of simple combinations and partitions of numbers, which does not rely on counting (Wright, Martland & Stafford, 2000). Examples of these combinations are the addition of two numbers in the range 1 to 5, doubles of numbers in the range 1 to 5 and

beyond. The process of partitioning, such as 8 is 4 and 4, 6 is 4 and 2, is the complement of combining. Children learn to provide answers almost immediately to questions such as $3 + 3$, using non count-by-ones procedures. Numerical knowledge of this kind has been referred to as “automised” or “habituated” (Wright, Martland & Stafford, 2000, p. 30) or “mastery” (Baroody, 2006, p. 22). Recent studies provides strong indications that children’s ability to habituate simple addition facts through combining and partitioning of small numbers can significantly facilitate development of advanced numerical strategies, that is non-count-by-ones strategies.

Treffers (2001) calls children’s ability to combine and partition numbers as “structuring numbers”. In structuring numbers, children mentally arrange numbers or put them in ways that can easily help them solve addition and subtraction of numbers. Children demonstrate by their ability to use known number facts like doubles, near doubles, near ten or using base-five or base-ten (5 or 10) as an anchor to solve addition and subtraction tasks. Ellemor-Collins and Wright (2009) also add that structuring numbers assists children to develop non-counting-by-ones strategies (facile strategies). A case study (Ellemor-Collins & Wright, 2008a) showed that appropriate instructional intervention methods and materials may promote children’s ability in structuring numbers and advance children’s strategies in solving decade addition from counting-by-ones to facile non-counting-by-ones strategies.

Zhang (1988) and Zhang and Liu (1991) call children’s ability to combine and partition numbers as ‘part-whole relations’, and assert that exposing learners to these intrinsic

links between part and whole is essential to mastering mathematical concepts. Lin (1994) argues that composition or decomposition of number is the core concept related to learning addition and subtraction. It contains four specific relationships: equal ($B = A + A'$); reverse ($B - A = A'$, $B - A' = A$); complementary [$B = A + A' = (A - n) + (A' + n) = \dots$]; and commutative ($B = A + A'$; $B = A' + A$), where A' and A are two subsets and B is the total. Zhang and Liu (1991) contend that the decomposition of the whole into parts and the combination of parts into the whole are determined by the nature of the relationships of including and being included. In other words, an understanding that the whole is greater than its parts and that the parts are each less than the whole promotes children's comprehension of the meaning of addition and subtraction.

So, when teachers expose children to the interrelationship between part and whole, and make explicit that mathematical knowledge is internally related and organized, children's thinking processes are enhanced (Zhou & Peverly, 2005). In this regard, Resnick (1983a) wrote:

Probably the major conceptual achievement of the early school years is the interpretation of numbers in terms of part and whole relationships. With application of a Part-Whole schema to quantity, it becomes possible for children to think about numbers as compositions of other numbers. This enrichment of number understanding permits forms of mathematical problem solving and interpretation that are not available to younger children (p. 114).

Studies have shown that if learners understand part-whole relationships, they are better able to understand the relationships among number, addition, and subtraction (Baroody, Ginsburg & Waxman, 1983; Fischer, 1990); attain the most advanced level of addition

and subtraction problem solving (Riley, Greeno & Heller, 1983); and understand place value (Resnick, 1983a). Place value refers to “the value of a digit which is dependent on its location within a number, such as units, tens, hundreds” (Cotton, 2013, p. 56). For instance, in 352, the place value of the 5 is ‘tens’ and the 5 is worth 50. These studies show that children invent their own strategies they use in solving number problems. In addition, the studies reveal that teachers can intervene and help learners advance their strategies of working with numerical concepts to more sophisticated levels. Teachers cannot meaningfully intervene unless they are aware of the strategies children use to deal with numerical situations so that they can build on them. Therefore, it is important to explore how children learn number concepts.

There are three developmental levels children pass through in learning single-digit additions and subtractions (Fuson, 1992b). At Level 1, children construct addition or subtraction situations using physical objects of some kind. These objects are used to model directly the addition or subtraction operation given in the situation. At a given moment, an object can first be part of an addend and can later be considered as part of the total or vice versa. Children count all the objects to add, and they take away and count the remaining objects to subtract.

At Level 2, children can simultaneously consider all three quantities in an addition or subtraction situation by embedding the addends within the total and considering objects as being simultaneously part of the addend and part of the total. Children can now count words in the number-word sequence instead of only counting objects, and they can

abbreviate the count of the first addend. Thus, to add, they can count on from one addend word while keeping track of the other addend words counted on, or they can add on by adding objects for one addend onto those for the other addend while counting on.

To subtract, they may count back from the total, keeping track of the addend counted back; count back from the total to an addend; or count up from the known addend to the total, keeping track of how many are counted up.

At Level 3, the addends no longer have to be embedded within the total but exist outside in a numerical triplet structure in which the two addends are seen as equivalent to the total. Quantities are composed of ideal chunkable unit items that can be combined and separated in flexible ways. A given numerical triplet can be recomposed into a related triplet. In this way, children can transform a given triplet with one unknown member into a triplet of known facts. To derive these facts for solutions, children commonly use doubles ($a + a$) in the United States, such as $7 + 6 = 6 + 6 + 1 = 12 + 1 = 13$. In Asian countries, children learn to recompose numbers into ten-structured triplets (Fuson, Stigler & Bartsch, 1986; Fuson & Kwon, 1992a). For example, $7 + 6 = 7 + 3$ (to make ten) $+ 3 =$ ten three (13). For subtraction, one can "take from ten" ($13 - 7$ is ten three - 7: take 7 from the ten is 3 plus the 3 in ten three is 6) or go down over ten (ten three - 7 is 3 down to ten and 4 more from the 7 goes down to 6). Such ten-structured methods are particularly useful in multidigit addition and subtraction, where each next larger multiunit is related by ten. In this way, ten-structured methods enable children to recompose ten or more of one multiunit into one next larger multiunit and the leftover of that multiunit, or recompose a larger multiunit into ten of the next smaller multiunit in order to subtract.

In addition, Fuson, Wearne, Hiebert, Murray, Human, Olivier, Carpenter, and Fennema (1997) argue that children eventually memorise many single-digit addition combinations. Because new facts are memorised during each of the three levels (for example, $2 + 2$ is learned very early), using a known fact is not really a special conceptual level. Rather, it occurs at all three levels. Children gradually learn more and more number combinations as known facts. Furthermore, Fuson et al (1997) add that children at Level 1, who can count above 10, can use a unitary multidigit conception to add two 2-digit numbers by making objects for each number and counting all of the objects. They can subtract by making objects, taking away from those objects, and counting the remaining objects. Children at Level 2 can count on by ones, add on objects by ones, or verbally count all by ones to add. To subtract, they can count back or count up to by ones. However, keeping track of the number counted on, up, or back may be difficult because it will be so large. These methods are constrained only by how high a child can count and keep track accurately.

In line with Fuson (1992b) and Fuson et al (1997), recent research also indicates that children typically progress through three phases in learning basic number combinations or a family of related number combinations—the single-digit addition combinations and their complementary subtraction combinations (Baroody, 2006; Baroody, Bajwa & Eiland, 2009). In phase 1, children use counting strategies, in which they make use of object counting such as using blocks, fingers and marks or verbal counting to determine an answer to a numerical problem. In phase 2, they base their calculations on reasoning strategies; that is, using or inventing known information like known number facts or

relationships to logically deduce the answer of an unknown number combination. In phase 3, children rely on mastery strategies. Mastery strategies are concerned with efficient -fast and accurate -production of answers from a memory network (Kilpatrick et al., 2001). As such, mastery strategies free cognitive resources so that attention can be focused on more complex matters. This ready access to number combination knowledge can facilitate problem solving and both mental and written computation with multi-digit and rational numbers (National Mathematics Advisory Panel, 2008).

These phases also relate to Askew (2013) who identifies three types of strategies children use in answering single-digit addition and subtraction calculations problems, namely counting, decomposing and retrieving. With counting strategies, children count all, count on from one of the numbers or count back from one of the numbers. With decomposition, they split one or both of the numbers to make retrievable number facts, such as $5 + 6 = 5 + 5 + 1$, yielding 11. Finally, with retrieval strategies, children can recall an answer from their memory within 3 seconds. In this regard, Askew (2013) adds that learners gradually progress from counting strategies, to decomposition strategies to retrieval strategies. In addition, he contends that proficiency implies selecting an efficient strategy on the part of the learner.

Counting and reasoning strategies are characterised by conscious or deliberate and, thus, relatively slow cognitive processes while mastery strategies are characterised by nonconscious or automatic and, thus, relatively fast cognitive processes (Baroody, Bajwa & Eiland, 2009). Mastery strategies can be achieved by either rote or meaningful

memorisation (Brownell, 1935). However, rote memorisation produces routine expertise, that is, knowledge that can be applied efficiently and appropriately to familiar tasks but not flexibly to new tasks (mastery with limited fluency). On the other hand, meaningful memorisation yields a rich and well-interconnected web of factual, strategic (procedural), and conceptual knowledge. The result is adaptive expertise, that is, well-understood knowledge that can be applied efficiently, appropriately, and flexibly to new, as well as familiar, tasks (mastery with fluency). Over the course of their development, children replace slow counting procedures and thinking strategies (inefficient reconstructive processes) with rapid fact retrieval (Ashcraft, 1982). Meaningful instruction (the teaching of thinking strategies) would probably contribute more directly to this process than a drill approach alone (Baroody, 1985). Children normally do not memorise and store all 400 or so basic combinations. In other words, children do not learn basic number combinations as so many separate entities or bonds (as hundreds of feats of memory) but as a system of interrelated experiences (Olander, 1931). Rules, procedures, and principles become routinised to make mastering the basic combinations a cognitively manageable task (Baroody, 1985).

The teaching of thinking strategies has been advocated to help children (a) learn numerical relationships and (b) foster the automatic recall of number facts (e.g., Brownell, 1935). To this end, Poincare (1905) argues that mastery with fluency “is built up of facts as a house is of stones, but a collection of facts is no more than a pile of stones is a house” (p. 141). In other words, as a framework and cement are necessary to transform a pile of stones into a house, number patterns and relations can serve to

structure and interconnect factual knowledge— to transform an otherwise amorphous bundle of facts into a wellorganised body of knowledge (Baroody, Bajwa & Eiland, 2009). So, it is important to identify and build on children’s strategies in number problems so that they achieve fluency in working with number concepts.

2.8 Place value and base-ten structures

Multi-digit knowledge includes knowledge of the numeration system and place value (Hiebert & Wearne, 1996). Thompson & Bramald (2002) distinguish quantity value from column value. For example, with quantity value, 47 is split into forty and seven, while with column value, it is split into 4 units of ten and 7 units of one. They argue that children’s mental strategies only depend on quantity value.

Researchers have charted learning trajectories from using counting-by-ones strategies, through increasingly powerful uses of units of ten and other base-ten structures (Ellemor-Collins & Wright, 2007). In a synthesis from four research projects, Fuson, Wearne, Hiebert, Murray, Human, Olivier, Carpenter and Fennema (1997) proposed a developmental sequence of children’s two-digit conceptual structures. The structures incorporate children’s relations among written numerals, number words, and quantities: unitary multidigit conception (53 as one, two, ...fifty-three); decade and ones conception (53 as one, two ... fifty; and ‘fifty’-one, ‘fifty’-two, ‘fifty’-three in which 53 is split into the decade and ones producing the incorrect ‘50’3 when writing it); sequence-tens and ones conception (53 as ten, twenty, ... fifty; and fifty-one, fifty-two, fifty-three); separate-tens and ones conception (53 as five tens and three ones); and integrated-

sequence-separate tens conception (53 as ten, twenty, ... fifty; and fifty-one, fifty-two, fifty-three and also as five tens and three ones). A sixth, incorrect conceptual structure was labelled concatenated single digit conception (53 as five and three).

Developing the work of Steffe and colleagues, Cobb and Wheatley (1988) distinguished three levels in which children construction of ten as a unit. The levels were evident in children's thinking in additive tasks. Children operating at level 1 manipulate ten units and one unit separately, and cannot coordinate them. At level 2, children can coordinate counts or collections of tens and of ones, in the context of representations of the quantities, but they cannot "simultaneously construct a numerical whole and the units of ten and one that compose it" (p. 7). Learners at level 3 can anticipate, without representations that a numerical whole consists of tens and ones units, and coordinate operations with these.

2.8.1 Sequence-based structures and strategies

When children begin to use base-ten structures in arithmetic, they develop a variety of multi-digit addition and subtraction strategies (Foxman & Beishuizen, 2002; Thompson & Smith, 1999). Sequence-based or jump strategies involve keeping the first number whole and adding (or subtracting) via a series of jumps, for example, $57 + 26$ as $57 + 10 = 67$, $67 + 10 = 77$, $77 + 3 = 80$, and then $80 + 3 = 83$. Collections-based or split strategies involve partitioning both numbers into tens and ones, and adding (or subtracting) separately with tens and ones, for example, $57 + 26$ as $50 + 20$ and $7 + 6$, then $70 + 13$, and then $(70 + 10)$ and 3 to get $80 + 3$ or 83 just as above.

Children's broad knowledge of number relationships and numeration is important for their mental computation (Heirdsfield, 2001). This includes knowledge of sequential structure: jumping by ten off the decade, locating numbers, number word sequences across decades, and making small hops (Fuson et al., 1997; Menne, 2001; Yackel, 2001). However, low-attaining children seem to use jump strategies less frequently and many do not develop knowledge of jumping in tens (Foxman & Beishuizen, 2002; Menne, 2001).

2.9 Number, number words and numerals

Drawing on Piaget's work, Kamii (2004) states that children's idea of number comes from logico-mathematical knowledge. Logico-mathematical knowledge consists of mental relationships—relationships originating in each person's mind (Kato, Honda and Kamii, 2006). Drawing on Piaget, Kato, Honda and Kamii (2006) distinguish five aspects of logico-mathematical relationships. The first is classification and involves mentally putting together things that are alike and separating those that are different (Inhelder & Piaget, 1964). The second, seriation, entails mentally ordering things according to their differences (Inhelder & Piaget, 1964). The third aspect is number (numerical relationships) while the fourth and fifth are spatial and temporal relationships. Studies into number, number words and numerals have focused on different areas. A research by Wright (1991) which investigated number knowledge possessed by children at the beginning of the kindergarten year also included an investigation of children's number word sequence development. A research by Young-Loveridge (1991) focused on the development of children's number concepts from the age five to nine.

More recent research studies have focused on children's understanding of numbers (Signe & Vicki, 2008; Olive, 2001).

2.10 Global perspectives on children's number knowledge

This section discusses studies related to children's learning of early number globally. It will do this by highlighting some studies done in Australia, United States of America, Indonesia, Malaysia, South Africa and finally Malawi.

In Australia, Gervasoni (2007) carried out a 3-year longitudinal study involving over 4000 children from 52 Catholic schools in the Ballarat Diocese, Victoria. The aim of the study was to assess the number knowledge of children when they first began school and over the first three years of school. Results were that in the counting domain, over half of the group knew the number word sequence to 20, and many of them could count a collection of at least 20 items. The remaining children could not correctly count the items because they were not yet familiar with number names and sequences to 20. However, some children could count forwards beyond 110 and backwards from 24, and others could skip count by 10s, 5s and 2s.

In the addition and subtraction strategies domain, cumulatively 7 percent of the children were able to use the count-on strategy to work out number problems, 34 percent used count-all strategy, while the remaining children were not able to solve such problems.

According to Grevasoni (2007), the findings highlighted the extent and diversity of children's number knowledge when they begin school and throughout the first three years of schooling.

In Malaysia, a study by Munirah, Rohana, Asrul and Ayminsyadora (2009) investigated primary school learners' mental computation strategies in addition and subtraction problems. The findings were that learners invented their own intuitive strategies when asked to solve problems using mental computation even when mental computation may or may not have been formally taught to them. Secondly, while some learners did invent their own intuitive strategies, there were others who did not display ability for mental computation. Although the study did not connect teachers' teaching strategies with learners' strategies, the findings from the study raised questions about whether learners do invent their own strategies or whether their use of intuitive strategies were indirectly encouraged by modeling teachers' own mental computation strategies.

In United States of America, Carpenter, Fennema, Franke, Levi and Empson (1998) conducted a 3-year longitudinal study involving 82 Grades 1 to 3 children. The aim of the study was to investigate the role that invented strategies may play in developing an understanding of multi-digit addition and subtraction concepts and procedures. The children were individually interviewed 5 times on a variety of tasks involving base-ten number concepts and addition and subtraction problems.

The results were that about 90% of the children used invented strategies and that the children who used invented strategies before they learned standard algorithms demonstrated better knowledge of base-ten number concepts and were more successful in extending their knowledge to new situations than were children who initially learned standard algorithms. Carpenter et al. (1998) concluded that children can invent strategies for addition and subtraction and that the results illustrated both what that invention affords and the role that different concepts may play in that invention.

In South Africa Mofu (2013) conducted an intervention programme using an out of school mathematics club with a small sample of five Grade 4 learners. The aim of the study was to investigate learners' multiplicative reasoning using the Mathematics Recovery (MR) Programme using a Learning Framework in Number (LFIN) for multiplicative reasoning. Data was collected using pre- and post- individual orally administered interviews.

Following the intervention programme implemented in just 4 weeks, the results were that 4 of the learners moved up by 1 step on the learning framework in number whereas the fifth learner moved up by 2 steps. According to the study, the findings showed an overall improvement in multiplicative proficiency for all learners. The study concluded that the MR programme was effective in the South African context despite that the period for the intervention was only four weeks.

In South Africa also, Wasserman (2015) conducted an action research which focused on recovery of early arithmetic strategies with one Grade 4 class of learners in a township school in Port Elizabeth in the Eastern Cape. The aim of the study was to understand the possibilities and constraints of the implementation of a Mathematics Recovery Programme (MRP) adapted from the widely implemented work of Wright et al. (2006, 2012). The MR programme was adapted and assessments and intervention were administered with groups of (rather than individual) learners with one class of 23 learners based on eight recovery sessions.

The findings showed some progress for all learners in terms of their early arithmetic strategies and conceptual place value. Although the need for a longer recovery period was acknowledged, the adapted MR programme enabled some progress in levels and stages of conceptual knowledge (as conceptualized by Wright et al.' (2006)'s Learning Framework in Number) for these two domains. The study, therefore, concluded with some reflections and recommendations for the future.

In Zambia, Young (2016) carried out a seven week after-school intervention programme with a class of Grade 2 learners aged seven to eight in a rural Zambian primary school. The aim of the study was to investigate the possibility of adapting the Mathematics Recovery Programme (MRP) for use in a whole class setting, and to research the effectiveness of such an adapted programme. The study also aimed at investigating the extent of the phenomenon of unit counting and other early arithmetic strategies used in the early years in Zambia.

The focus of the study was on the early arithmetic strategies aspect of the Mathematics Recovery programme. Data was collected from a sample of 6 learners using video recordings. Although limited by time and research focus, the study found that all learners made some progress in early arithmetic strategies, and indicates that the Mathematics Recovery programme has potential for adaptation for early intervention in whole class teaching to address the mathematical education challenges in Zambia and beyond. The study also found that unit counting predominated among the selected learners, but that strategies were not yet entrenched, indicating that this was a suitable age for early intervention. Although these studies were intervention programmes, the results indicate that learners learn meaningfully when they are taught in ways that take into account what they are able to do. Therefore, exploring how children in a Malawian primary school learn number concepts is one step towards building on what the learners already know for effective teaching and learning of early number concepts.

2.11 Studies carried out in Malawi

A study by Mazombwe-Kutsaila (2011) was done in Malawi in four pre-schools (2 urban and 2 rural). The aim of the study was to investigate teaching and learning practices that caregivers provided to enhance the development and acquisition of early numeracy skills in pre-school children. The findings were that in all the pre-schools, children were engaged in counting, writing, adding, and subtracting small numbers. In addition, two of the pre-schools, incorporated modeling in counting, addition and subtracting of numbers. However, this study was conducted in pre-schools and targeted caregivers. Therefore,

little is known about how children in lower primary school in Malawi acquire number concepts. This study is therefore important as it aimed at reducing that gap.

In 2010, the Malawi Teacher Professional Development Support (MTPDS) in conjunction with the United States Agency for International Development (USAID) under the Early Grade Mathematics Assessments (EGMA) conducted a baseline study. The study was aimed at investigating the level of mathematics skills of children in Malawi in order to obtain a national level perspective of early numeracy development, to provide baseline data for USAID-Funded MTPDS Programme activities, and identify areas of weakness for future decision-making and curricular and pedagogical interventions (Brombacher, 2011). The study included 50 schools across all six education divisions in Malawi, and involved a total of 999 standard 2 and standard 4 learners.

The overall results showed a quite clear difference in the performance of the standard 2 and the standard 4 learners in which the top 25% of the standard 2 learners performed as well as the bottom 25% of the standard 4 learners. According to Brombacher, this difference could suggest that, in general, children in Malawi were benefiting from attending school. However, the learners' performance was generally low when compared to both the expectation of the Malawi curriculum for mathematics and the international literature on expectations of performance by children in these standards. In addition, findings especially on addition and subtraction problems were that nearly 56% of standard 2 learners were unable to answer even single-digit addition sums with an answer

less than 10 correctly and nearly 65% of them were unable to answer single-digit subtraction differences correctly.

On word problems, both the standard 2 learners and the standard 4 learners marginally underperformed the overall test average on the word problem subtest (20.6% versus 21.6% for standard 2 learners and 60.5% versus 61.8% for standard 4 learners). The study recorded whether learners used fingers, counters, and/or paper and pencil in working with given number problems. However, it did not take note of how learners used these materials to aid solve the numerical problems. Several other studies (Kunje, Selemani-Meke & Ogawa, 2009; MoEST, 2010) have investigated factors that have led to learners' underachievement in mathematics in Malawi. However, none of them has adequately explored children's learning of number in Malawi. As such, little is still known about learners' acquisition of number in lower classes in Malawian primary schools.

All the studies discussed in this chapter inform this study on different aspects of teaching and learning of number. Although many of the studies were conducted in developed countries whose context is different from Malawi, the findings can be related with understanding of the Malawi context and lessons learnt.

2.12 Conceptual framework

This study was informed by a conceptual framework called Learning Framework in Number (LFIN), which was developed for the Mathematics Recovery (MR) Programme by Bob Wright in Australia in the early 1990s (Wright, Martland & Stafford, 2006).

LFIN drew on research in early number conducted by Steffe and Cobb (1983), Cobb and Wheatley (1988) and Wright (1989, 1991a, 1991b), and uses a model of stages of learning early numeracy which are classified into three strands (Cobb & Wheatley, 1988). Details of the the conceptual framework are discussed below.

2.12.1 Learning framework in number (LFIN)

The LFIN is organised into four parts, divided into eleven aspects of children's early numerical knowledge.

Table 1: Learning framework in number (LFIN)

Part A	Part B	Part C	Part D
1. Early arithmetical strategies	1. Forward number word sequences and number word after	1. Structuring numbers 1 to 20 (a) combining and partitioning	1. Early multiplication and division
2. Base-ten arithmetical strategies	2. Backward number word sequences and number word before	(b) Spatial patterns and subitizing (c) Temporal sequences (d) Finger patterns	
	3. Numeral identification	(e) Five-based (quinary-based strategies)	

Source: Wright, Martland and Stafford (2006, p. 20)

Part A of the LFIN consists of stages for early arithmetic learning (SEAL) and base-ten strategies (BTS). Part B contains forward number word sequences and number word after (FNWSs and NWA), backward number word sequences and number word before (BNWSs and NWB), and numeral identification (NI). Part C has structuring numbers 1 to 20, and finally, part D comprises early multiplication and division (Wright, Martland & Stafford, 2006). All the eleven aspects of the LFIN are considered important in children's early numerical learning and teachers using the LFIN are able to develop teaching strategies that allow children simultaneously to work and progress on more than one part of the framework. This study considered six aspects of the LFIN. These are stages for early arithmetic learning (SEAL), forward number word sequence (FNWS), number word after (NWA), backward number word sequence (BNWS), number word before (NWB), and numerical identification (NI). Each of these six aspects is described briefly in the next sections.

2.12.1.1 Stages for early arithmetic learning (SEAL)

The stages for early arithmetic learning (SEAL) is considered the most important aspect of the learning framework in number (Wright, Martland & Stafford, 2000). The SEAL sets out a progression of the strategies children use in early numeracy situations they find problematic to them. For example, situations that require children to find how many objects there are in a collection, and others that require addition and subtraction. The SEAL model was adapted for the MR programme from the theory of children's counting types developed by Steffe and colleagues such as Steffe (1992a), Steffe and Cobb (1988) and related research by Wright (1989, 1991a). Table 1 gives a model of the stages for

early arithmetic learning which the study used. In testing the viability of the theory of children's counting types further, Wright (1989; 1991a) found that there are younger children who cannot count visible objects in the range 1 to 10 because they have not yet acquired number names in the range 1 to 10 or because they cannot coordinate number names with objects as they count them in this range. Wright (1989) labelled these children 'emergent counters', and added this category to the model.

Table 2: Model of stages for early arithmetic learning (SEAL)

Stage	Name of Stage	Characteristic
0	Emergent	Child cannot count visible items.
1	Perceptual	Child can count visible items only.
2	Figurative	Child can count invisible items but starts from 1.
3	Advanced counting-by-ones	Child can count invisible items, using a count-on strategy to solve addition or missing addend tasks, and may use a count-back strategy (count back-from or count-back-to) to solve subtraction, missing subtrahend or removed items tasks.
4	Facile	Child counts invisible items quickly or uses a non-counting-by-ones strategy.

Source: Wright, Martland and Stafford (2006, p. 22)

The ‘Stage 0’ is used for children who have not attained the first stage of the SEAL model. Developing facile mental strategies for addition and subtraction involving 2-digit numbers is a critically important goal of arithmetic learning (Wright, Martland & Stafford, 2006). It lays a strong foundation for all further learning of arithmetic (Wright, Ellemor-Collins & Lewis, 2007).

2.12.1.2 Numeral identification (NI)

Learning to identify, recognize and write numerals is an important part of early mathematical literacy development and is equally important for numerical development (Wright, Martland & Stafford, 2000). Wright, Martland and Stafford (2000) make a distinction between the terms “identify” and “recognising”. To identify is “to state the name of a displayed numeral” whereas recognising is “the complementary task of selecting a named numeral from a randomly arranged group of displayed numerals” (p. 29). Table 2 outlines a progression of four levels of children’s development of numerical identification which the study explored.

Table 3: Model for the development of numeral identification

Stage	Name of Stage	Characteristic
0	Emergent Numeral identification	Cannot identify some or all numerals in the range 1 to 10.
1	Numerals to 10	Can identify numerals in the range 1 to 10.
2	Numerals to 20	Can identify numerals in the range 1 to 20.
3	Numerals to 100	Can identify one- and two-digit numerals.

Adapted from: Wright, Martland and Stafford (2006, p. 24)

The label ‘Level 0’ is used for children who have not yet attained the first level.

2.12.1.3 Forward and backward number word sequences

The term ‘number words’ refers to “the spoken and heard names of numbers” (Wright, Martland & Stafford, 2000, p. 27). Steffe and Cobb (1988) made a distinction between counting and reciting a sequence of number words. The term ‘counting’ is used only in cases that involve co-ordination of each spoken number word with an actual or imagined or conceptualised item. In this case, counting typically occurs in situations that are problematic for children, like when they are working out an addition or subtraction problem or determining the numerosity of a collection of objects (Wright, 2013).

On the other hand, the activity of merely saying a sequence of number words is not referred to as counting.

2.12.1.3.1 Forward number word sequences (FNWSs)

The term ‘forward number word sequence’ (FNWS) refers to “a regular sequence of number words, for example, the number words from fifteen to twenty-three said or considered in a forward direction” (Wright, 2013, p. 26). Such a sequence is typically but not necessarily by ones; for example, the FNWS from fifty-seven to seventy-two, and the FNWS by fours from six. FNWS is regarded different from counting, in the sense of counting to determine the number of objects in a collection. In addition, though some authors use the term “verbal counting” or “rote counting” for FNWS, Wright (2013) argues that “rote counting” is unsuitable because experience of closely observing many children in the activity of saying FNWSs shows that there is more to saying FNWS than simply doing “rote counting”.

This study was aimed to explore children’s ability to make FNWSs and later determine children’s levels using the FNWS model given in Table 3. The label ‘Level 0’ is used for children who have not reached the first level.

Table 4: Model for the construction of forward number word sequences (FNWS)

Stage	Name of Stage	Characteristic
0	Emergent FNWS to 10	The child cannot produce FNWS from 1 to 10.
1	Initial FNWS up to 10	The child can produce FNWS from 1 to 10. The child cannot produce the number word just after a given number word in the range 1 to 10. Dropping back to one does not appear at this level. Children at levels 1, 2 and 3 may be able to produce FNWS beyond 10.
2	Intermediate FNWS up to 10	The child can produce the FNWS from 1 to 10. The child can produce the number word just after a given number word but drops back to 1 when doing so.
3	Facile with FNWS up to 10	The child can produce the FNWS from 1 to 10. The child can produce the number word just after a given number word in the range 1 to 10 without dropping back. The child has difficulty producing a number word just after a given number word, for numbers beyond 10.
4	Facile with FNWS up to 30	The child can produce the FNWS from 1 to 30. The child can produce the number word just after a given number word in the range 1 to 30 without dropping back. Children at this level may be able to produce FNWSs beyond 30.
5	Facile with FNWSs up to 100	The child can produce FNWSs in the range 1 to 100. The child can produce the number word just after a given number word in the range 1 to 100 without dropping back. Children at this level may be able to produce FNWSs beyond 100.

Source: Wright, Martland and Stafford (2006, p. 23)

2.12.1.3.2 Backward number word sequences (BNWS)

The term ‘backward number word sequence’ (BNWS) refers to “a sequence of number words, for example the sequence from twenty-three to fifteen said or considered in a backward direction” (Wright, 2013, p. 26). BNWS is also different from a rote activity just as with saying FNWSs. This study was aimed to explore children’s ability to say BNWSs and thereafter determine the levels they had attained in this regard. Table 4 below was used for this purpose. The label ‘Level 0’ is used for children who have not attained the first level.

Table 5: Model for the construction of backward number word sequences (BNWS)

Stage	Name of Stage	Characteristic
0	Emergent BNWSs to 10	The child cannot produce BNWS from 10 to 1.
1	Initial BNWS up to 10	The child can produce BNWS from 10 to 1. The child cannot produce the number word just before a given number word in the 10 to 1. Dropping back to one does not appear at this level. Children at levels 1, 2 and 3 may be able to produce BNWS beyond 10.
2	Intermediate BNWS up to 10	The child can produce the BNWS from 10 to 1. The child can produce the number word just before a given number word but drops back to 1 when doing so.
3	Facile with BNWS up to 10	The child can produce the BNWS from 10 to 1. The child can produce the number word just before a given number word in the range 10 to 1 without dropping back. The child has difficulty producing a number word just before a given number word, for numbers beyond 10.
4	Facile with BNWS up to 30	The child can produce the BNWS from 30 to 1. The child can produce the number word just before a given number word in the range 30 to 1 without dropping back. Children at this level may be able to produce BNWSs beyond 30.
5	Facile with BNWSs up to 100	The child can produce BNWSs in the range 001 to 1. The child can produce the number word just before a given number word in the range 100 to 1 without dropping back. Children at this level may be able to produce BNWSs beyond 100.

Source: Wright, Martland and Stafford (2000, p. 24)

2.12.1.4 Number word after (NWA)

Number word after (NWA) task assesses the child's ability to say the number word after a given number word. The aim is to observe whether the child mentions the next number word immediately or soon after being asked; says the number word sequence forward from one aloud or subvocally (dropping back strategy); or is unable to answer (Wright, Martland & Stafford, 2000). The numbers are presented in a random order within the ranges of 1 to 10, 11 to 30 and 30 to 100 while accommodating numbers with 9 or 0 in the ones place. However, for this study, the numbers were purposively arranged. The study used the model for FNWSs (Table 3) for determining children's levels in NWA.

2.12.1.5 Number word before (NWB)

Number word before (NWB) task assesses the child's ability to say the number word before a given number word. They are aimed at checking whether the child mentions the next number immediately or soon after being asked, says the number word sequence forward from one aloud or subvocally, or is unable to answer (Wright, Martland & Stafford, 2000). As is the case with NWA, numbers are presented in random order (purposively arranged for this study) and in the ranges 1 to 10, 11 to 30, and 30 to 100 with particular interest in numbers ending in 0 or 1 in the ones place. The model for BNWSs (Table 4) was used for determining children's levels in this task.

2.13 Chapter summary

First sections of this chapter have reviewed some of the available literature on the teaching for number sense, children's acquisition of number and children's thinking strategies. The last section of the chapter has outlined the conceptual framework which guided the understanding of issues in all chapters of the thesis. The next chapter discusses the methodology used in this study.

CHAPTER 3

METHODOLOGY

3.0 Chapter overview

This chapter discusses the research design and methodology of the study. It also describes and justifies the instruments that were used to collect data. It further describes the sample and sampling procedures, how data was analysed and the methods that were used to ensure trustworthiness of the study. The chapter concludes by describing how ethical issues were adhered to.

3.1 Research approach

The study used a qualitative research approach as a framework in which to answer the research questions, collect, analyse and interpret data. According to Maxwell (2013, p. 30), qualitative research is concerned with “understanding the process by which events and actions take place”. Merriam (1988, p. xii) also stated that “the interest [in a qualitative research] is in process rather than outcomes”. This, however, does not mean that qualitative research is not concerned with outcomes. Rather, it emphasises more on getting the processes that led to such outcomes (Maxwell, 2004a, 2004c; Patton, 1990). Furthermore, O’Leary (2010, pp. 113-114) argues;

Qualitative research is concerned with “depth over quantity and works at delving into social complexities in order to truly explore and understand the interactions, processes, lived experiences, and belief systems that are a part of individuals, institutions, cultural groups, and even the everyday.

For this reason, O’Leary further argues that;

Delving into qualitative methodologies therefore means working in a world that accepts and even values: the search for holistic meaning; research conducted in natural settings; ...; small numbers; non-random sampling strategies; rich qualitative data; inductive analysis; idiographic interpretation; and even the possibility of negotiated outcomes that recognise the need for the researched to be party to a researcher’s constructed meanings... (p. 114).

This is achieved by collecting rich descriptive data from what is seen, heard, and understood (Bogdan & Bicklen, 2007; Creswell, 2009).

The study was aimed at exploring children’s learning of number in lower classes in Malawian primary schools. As such, it was mainly concerned with getting processes that children used to acquire these concepts. For this reason, the research focus was especially suited for a qualitative research that helped the researcher in collecting rich data from what was seen, heard and understood through observations, interviews and analysis of documents (schemes and records of work, lesson plans, classroom wall charts and exercise books) in order to holistically capture learners’ conceptions and strategies about the concepts under study.

In a qualitative research tradition, knowledge claims are made based on a constructivist perspective (O’Leary, 2010; Creswell, 2009). In this regard, qualitative research was

more convenient for this study because it “works at accepting multiple realities through the study of a small number of in-depth cases” (O’Leary, 2010, p. 105). In this regard, methodologies that were used to generate knowledge and understandings allowed the research participants to give their versions of knowledge construction and understandings.

3.1.1 Research design

Within the qualitative research paradigm, a case study design was used. A case study is a “method of studying elements of the social through comprehensive description and analysis of a single situation or case” (O’leary, 2010, p. 174). Since the study dealt with young children, a case study is a useful tool for studies of that nature (Mukherji & Albon, 2010). The case study approach helped the study came up with profiles of each child’s learning of number.

In addition, a case study generates rich information which allows for an in-depth understanding of the people and/or context under study (Stake, 2000). This is achieved by using multiple instruments in collecting data to have a holistic picture of the issue under study (Mukherji & Albon, 2010). To achieve this, the study used one-to-one interviews, lesson observation schedules, and analyses of documents such as schemes and records of work, lesson plans, classroom resources and children’s exercise books.

However, case studies have some limitations. Since case studies are an in-depth study of one or very few cases, the findings cannot be generalised to a population as a whole

(Gilbert, 2008a). This, however, is not much of a worry since the aim of conducting a case study is to get a deeper understanding of the issue under study and not generalising its findings (Mukherji, & Albon, 2010). For this study, the aim was to explore children's learning of number, particularly counting, addition and subtraction. So, studying a few cases still shed light on individual children's acquisition of number concepts which resulted in sound implications and recommendations.

Finally, the nature of case study research means that the findings may not be easily replicable by others (Mukherji & Albon, 2010). The researcher may bring bias to the proceedings by being selective in the case they decide to study as well as in what and how they record, and in analysing the data. If not careful, it is easy to see what the researcher wants to see. However, qualitative research is contextual. As such, issues of replication fall outside the scope of qualitative research tradition. To reduce researcher bias to the research findings, Creswell (2009) advises to triangulate data to bring about objectivity in a study.

3.2 The role of the researcher

Before joining teacher training colleges (TTCs) as a primary school teacher educator in 2016, the researcher had been teaching mathematics at secondary school for 12 years. While at secondary school, he had also served as a divisional trainer in mathematics for 7 years under the strengthening of mathematics and science in secondary education (SMASSE) initiative. SMASSE intervention uses ASEI (Activity Student Experiments Improvisation) movement and PDSI (Plan, Do, See and Improve) approach. With the

ASEI movement and PDSI approach to instruction, emphasis on thorough lesson planning, then implementation while observing how well the implementation is being done and making improvements in subsequent lessons on the part of the teacher. On the part of students, theirs is do activities and experiments in the lesson using improvised materials where conventional ones are in short supply. The belief is that students understand mathematical and science concepts well when they are given opportunities to interact with content through hands-on and minds-on activities. Hands-on experiences entail the activities and experiments students carry out in the lesson whereas minds-on activities entail the reasoning behind the activities and experiments students carry out in the lesson. In addition, no lesson in SMASSE is believed to be perfect. As such, it can be improved with adequate reflections from previous ones.

The researcher was aware that being a teacher educator would change the general behaviour of the mathematics teachers who took part in the study. Among other things, the teachers would think that they were being supervised or evaluated not observed which would make them teach in superficial way in order to impress the researcher. Creswell (2009) also adds that the researcher's previous experiences may bring certain biases to a study. These biases may shape the way the researcher views and understands the data collected and the way they interpret their previous experiences. To reduce these biases, the researcher explained clearly the aim of the study and tried as much as possible to take off the researcher's position as a teacher-educator and to be at par with the teachers. Added to that, the researcher brought about objectivity by triangulating the data sources

and the data themselves. Finally the researcher stucked onto the use of data collection instruments in order to remain focused on the aim of the study.

3.3 Selection of study site and research participants

Decisions about where to conduct one's study and who to include in it (what is known as sampling) are an essential part of a given study's methods (Maxwell, 2013). According to Maxwell (2013), the term "sampling" is problematic for qualitative research, because it connotes a purpose of "representing" the population sampled, which is the usual goal of sampling in quantitative research. This is achieved by selecting settings and research participants by means of probability or convenient sampling. In qualitative research, Maxwell (2013) argues that the typical way of selecting settings and research participants is neither probability sampling nor convenient sampling. It falls into a third category called purposeful selection (Light et al., 1990) or purposive sampling (Palys, 2008). With purposive sampling, "particular settings, persons or activities are selected deliberately to provide information that is particularly relevant to your questions and goals, and that can't be gotten as well from other sources" (Maxwell, 2013, p. 97). To achieve this, a researcher uses personal judgement to select a sample, based on previous knowledge of a population and the specific purpose of the research (Fraenkel & Wallen, 2009). As such, purposive sampling enables researchers to choose a case because it possesses features that they are interested in (Silverman, 2005).

The present study focused on children's learning number in lower classes in Malawian primary schools. As such, learners were purposively selected and included those that

would show progress in learning number among other attributes. The researcher selected the learners in consultation with their mathematics teachers to ensure that those selected had attributes that were generally consistent with the aim of the study.

3.3.1 Study site

The study was conducted at one rural public primary school in Balaka district, which is in the South East Education Division (SEED). The school is located approximately 17 kilometres North-East of Balaka town. It was opened in 1952 and has Standards 1 to 8. At the time of this study, it had 20 teachers of whom 12 were male and 8 were female. The total enrolment of the school was 1616 (825 boys and 791 girls).

This school was selected for study because of its accessibility to the researcher. At the time of this study, Standard 1 had two streams, A and B. Similarly, Standard 2 had two streams, A and B. The class enrolment for Standard 1A was 234 (131 boys and 103 girls) and that for Standard 1B was 325 (145 boys and 180 girls). These learners were aged from 5 to 14 years. The class enrolment for Standard 2A was 152 (81 boys and 71 girls) and that for Standard 2B was 149 (79 boys and 70 girls). The learners' ages ranged from 6 years to 14 years. This shows that both Standards 1 and 2 classes were too large for a healthy teaching and learning in terms of classroom space and teacher-pupil ratio among other resources.

3.3.2 Research sample

The study sample included learners from Standards 1 and 2 as well as mathematics teachers in these classes. A total of 12 learners (6 learners from Standard 1 and 6 others from Standard 2) were purposively selected into the sample. To ensure anonymity of these learners, they were coded Learner 1, 2, 3... up to 12. Learners 1 to 6 were selected from Standard 1 while Learners 7 to 12 were selected from Standard 2.

Standard 1 learners were aged from 5 to 10 years while those from Standard 2 were aged from 10 to 13 years.

There were 5 mathematics teachers who were teaching Standards 1 and 2. All the 5 teachers were selected into the sample because they were knowledgeable about children's learning of number in those Standards. For the sake of anonymity of the teachers, they were coded Teacher A, B, C, D and E. Teachers A, B and C were selected from Standard 1 (Teacher A from Standard 1A while Teachers B and C from Standard 1B) whereas Teachers D and E were selected from Standard 2 (Teacher D from Standard 2B and Teacher E from Standard 2A). Initially, Teacher B was the one teaching mathematics in Standard 1B and by the time this study was conducted, Teacher C had just taken over the subject from Teacher B. So, the researcher included both of them in the sample.

Out of 5 teachers, 3 were female and the other 2 were male. Four of the five teachers were aged from 33 to 38 years while the other one was close to 50 years old. Four of the teachers had work experience of up to 5, mathematics teaching experience of up to 5 years, and mathematics teaching experience in infant classes (Standards 1 and 2) of up to

3 years. The fifth teacher had over 20 years teaching experience, over 15 years mathematics teaching experience, and had been teaching mathematics in infant classes for over 15 years as well.

3.4 Data collection instruments

The study used a number of instruments to collect data. These included separate interview guides for mathematics teachers and learners, lesson observation guide for both teachers and learners and document analysis guides for analysing teachers' and learners' documents; namely schemes and records of work, lesson plans, classroom resources and learners' exercise books. The instruments were first checked and approved by the researcher's supervisor before they were piloted and later modified and used for the main data collection.

3.4.1 Interviews

Mukherji and Albon (2010, p. 118) define research interview as “a method where one person asks questions of an individual or group of people with the expectation of getting answers to a particular question or an elaboration of their views on a particular topic”. For this study, face-to-face interview was used. In a face-to-face interview, the researcher engages research participants on a one-to-one basis (Creswell, 2009). For teachers, the study used a semi-structured interview guide (Appendix E) which consists of both closed and open-ended questions (Fraenkel & Wallen, 2009). Semi-structured interview loosens participants to respond to interview questions without unnecessary restrictions (Cohen, Manion & Morrison, 2007). Added to this, the researcher has room for probing for

clarifications where it was necessary. To develop the interview guide for teachers, the researcher adapted some questions from literature and also developed others guided by the research questions. Some questions (adapted from Susuwele-Banda, 2005, p. 153) were on teachers' experiences in teaching (general), mathematics teaching (in general) and mathematics teaching in lower primary school. Other questions targeted areas related to teachers' strategies in teaching early number concepts, resources used, learners' understanding of early number concepts and challenges faced by both teachers and learners among others.

For the learners, the study used an oral assessment interview guide (Appendices F and G). Questions on the interview guide were translated to Chichewa -the learners' most common vernacular language and language of instruction in Standards 1 to 4 at the school. The translation was done to be in tandem with the Malawi's 1996 language in education policy (LiEP) directive which requires that learners in lower primary (Standards 1 to 4) be taught in their mother tongue or vernacular as a medium of instruction (MoESC, 1996).

The clinical interviews helped the researcher observe children as they solved numerical problems to determine the strategies they used and any misconceptions they made (Gervasoni & Sullivan, 2007). In addition, the clinical interviews offered the researcher an opportunity for observational listening (Mitchell & Horne, 2011) and to probe children's mathematical understanding through thoughtful questioning when his observations failed to shed more light (Wright, Martland & Stafford, 2000). The

researcher developed the oral assessment interview guide for learners by partly adapting some from literature and partly developing others guided by the research questions. Questions for oral counting task, one-to-one correspondence counting task and addition and subtraction tasks were adapted from the Malawi Early Grade Mathematics (EGMA Malawi) National Baseline Report (2010) and the United States Agency for International Development (USAID) Early Grade Mathematics EDDATA II (2008). This was done bearing in mind the mathematics syllabi for Standard 1 and Standard 2. The other questions for number word sequences and numeral identification were developed following guidelines given provided by Wright, Martland and Stafford (2006) such as inclusion of special numbers like decades, doubles and those with a 0 or 1 in the ‘ones’ column.

3.4.2 Observations

Observation is concerned with a systematic process of recording patterns of behaviour of research participants without necessarily questioning them (Meree, 2007). More specifically, the study used a complete observer. This is a type of observation in which the researcher observes participants under study without taking part in the activities of the research participants (Creswell, 2009). This study engaged mainly young children some of whom might not have developed full communication skills needed to give an interview. So, observing them in a lesson helped the researcher to collect data which the children might not be able to provide otherwise (Mukherji & Albon, 2010). For mathematics teachers, observations enabled the researcher to collect data on teachers’ actual classroom practices and add it to the data collected from interviews.

Moreover, interview questions could not cover every aspect about the way teachers taught the concepts under study. So, these observations helped triangulate data.

To carry out these lesson observations, the researcher used a lesson observation guide (Appendix H) which was developed by the researcher guided by the research questions. Among other areas, interest was on teaching and learning strategies used for early number concepts, types of resources and how they were used, types of questions teachers posed during lessons and nature of lesson activities and experiences included in mathematics lessons.

3.4.3 Using documents and other visual texts

Macdonald and Tipton (1993) argue that documentary analysis is done in an attempt to triangulate a research study. According to Maxwell (2013, p. 103), triangulation is concerned with “using different methods as a check on one another, seeing if methods with different strengths and limitations all support a single conclusion”. In this case, triangulation reduces the risk that the researcher’s conclusions will reflect only the biases of a specific method, and allows the researcher to gain a more secure understanding of the issues being investigated. In addition, documents contain data that are thoughtful in that research participants have given attention compiling them (Creswell, 2009). However, with documents, not all people are equally articulate and perceptive.

Types of documents that might be analysed in a research study include newspapers and magazines, records, letters and memos, diaries, and government publications and official

statistics (Denscombe, 2003). However, in this research, the researcher analysed schemes and records of work, lesson plans, classroom resources and children's exercise books. To effectively carry out the document analysis exercise, the research used separate document analysis guides for mathematics teachers and learners (Appendices I and J). The document analysis guides helped the researcher capture teaching and learning strategies for early number concepts, resources used and learners' understanding of early number concepts. Table 5 gives a summary of the study's research questions, data sources, data collection methods and data type for each research question. As summarised in Table 5, the types of data collected in this research included teaching and learning strategies for early number concepts, children's conceptions of these concepts and learners' levels in basic numeracy. These data were collected from teachers, learners and teachers' schemes of work, lesson plans, classroom resources and learners' exercise books, which were collected through interviews, oral assessment, lesson observation and analyses of documents.

Table 6: Summary of research questions, data collection methods and instruments, data sources and type of data collected

Research Question	Data Type	Data Source	Data Collection Method
1. What strategies do mathematics teachers use to teach early number concepts (counting, addition, and subtraction)?	.Teaching strategies	.Teachers .Documents (lesson plans, schemes and records of work, charts)	.Observation .Interview .Document analysis
2. How do learners understand number concepts (counting, addition and subtraction)?	.Children's conceptions	.Teachers .Learners .Documents (learners exercise books)	.Observation .Interview .Document analysis
3. What strategies do learners use to count, add and subtract numbers?	.Children's learning strategies	.Learners .Teachers .Documents (learners' exercise books)	.Observation .Interview .Document analysis
4. What levels have learners reached in basic numeracy (numeral identification, forward and backward number word sequences, number word after and number word before)?	.Learners' levels of basic numeracy	.Learners .Documents (learners' exercise books)	.Observation .Interview .Document analysis

3.5 Data analysis and interpretation

Data analysis is a way of summarising and describing research findings, and then seeing if the researcher can find common patterns or themes (Roberts-Holmes, 2005). Doing this, therefore, allows the researcher to understand the topic under study in greater depth (Mukherji & Albon, 2010). To analyse and interpret the data collected, the researcher used thematic analysis. According to Riessman (2004b), thematic analysis emphasises on what the research participant said and not necessarily how they said it.

The data analysis was done following six steps as given by Creswell (2009). First the raw data was prepared for analysis by transcribing tape recorded interviews and video-recorded classroom observations, typing and expanding all field notes the researcher took on observations made during the data collection exercise. Transcribing was done by translating data from oral language, with its own set of rules, to a written language, with another set of rules (Cohen et al., 2000). Thereafter, the researcher listened to the audio and video tapes while reading and comparing them with the written text to make sure that what each research participant said matched with the written text.

Secondly, the data was read through over and over again to gain a general sense of the data and to reflect on its overall meaning. Rossman and Rallis (2003) point out that the process of reading the data again and again enables the researcher to be familiar in intimate ways with what they have learned. So, reading the transcripts and also listening to the audio and video tapes over and over again gave the researcher a chance to get familiar with major issues that arose from the data. In this regard, Gall, Borg and Gall

(2003, p. 453) add that in order to make meaning out of the data, the researcher needs to “examine case study data closely in order to find constructs, themes and patterns that can be used to describe and explain the phenomenon being studied”. This step helped the researcher to identify common patterns emerging in each of the transcripts of data by comparing responses participants gave for each question asked during interviews, and groups of words, phrases, questions or sentences that indicated particular strategies used in the teaching and learning of number concepts.

The common patterns the researcher identified across interviews and lessons observed, pointed to specific themes or categories. Bryman (2008) argues that identification of themes needs the researcher to look for recurring ideas and topics in the data. This is specifically achieved by looking for repetitions, indigenous typologies or categories, metaphors and analogies, transitions, similarities and differences, linguistic connectors, missing data, and theory related material (Ryan & Bernard, 2003).

Third, the researcher coded the data. Coding was concerned with “organising the material into chunks or segments of text before bringing meaning to the information” (Rossman & Rallis, 1998, p. 171). In addition, coding involved taking text data or pictures gathered during data collection, segmenting sentences (or paragraphs) or images into categories, and labelling those categories with a term. This term was often based in the actual language of the participants (called an *in vivo* term). The researcher did this by searching for and grouping together all responses to a question that emerged from the data on each research question and expressed the same idea. These categories became the bases for

coming up with themes guided by the research questions (Bryman, 2008; Ryan & Bernard, 2003). Later, the themes which emerged from various categories were compared and contrasted in order to check if there were similarities, differences and general trends. Fourth, the researcher used the coding process to generate a description of the setting or people as well as categories or themes for analysis. The themes that emerged from the data later became the bases for making an interpretation or meaning of the data (Creswell, 2009; Denzin & Lincoln; 1998).

Fifth, the step given above was followed by advancing how the description and themes would be represented in the qualitative narrative like a discussion by means of tables, visuals, figures as adjuncts to the discussions.

Finally, the researcher made an interpretation or meaning of the data. Interpretation, in this regard, simply meant making sense of findings, attaching meaning, offering explanations and drawing conclusions on the same (Denzin & Lincoln; 1998). The analysis of data was done in two phases.

Maxwell (2013) summarises the thematic process that was used to analyse the data in the following five steps:

The process of reflective qualitative data analysis requires researchers to: (1) organise their raw data; (2) ... code the data; (3) search for meaning through thematic analysis; (4) interpret meaning; and (5) draw conclusions – all this while keeping the bigger picture, i.e. research questions, aims and objectives, methodological constraints, and theory, clearly in mind (p. 257).

Data from documents such as teachers' schemes and records of work and lesson plans, learners' exercise books and classroom resources were analysed using document analysis guide. Cohen and Manion (1990) assert that document analysis is concerned with careful identification of appropriate category and unit of analysis. This meant working through the selected documents to identify strategies that both teachers and learners used in the teaching and learning of number concepts under study.

3.6 Credibility and trustworthiness of the study

Credibility, in a qualitative research process, is concerned with whether a research document is free from distortion and error (Macdonald & Tipton, 1993). Therefore, four ways were used to ensure credibility and trustworthiness of this study. These were triangulation, pilot testing of the data collection instruments, use of direct quotations from the interview data and/or actual work from the research participants, and use of respondent validity. These are discussed in the subsections that follow.

3.6.1 Triangulation

To triangulate, is to "obtain confirmation of findings through convergence of different perspectives. The point at which the perspectives converge is seen to represent reality" (Kasunic, 2005, p. 15). This is achieved by using different sources, different methods, different researchers, and different theories (Denzin & Linkon, 1994). The use of different perspectives in a single study serves as a check on one another and the researcher establishes whether these perspectives, despite having different strengths and weaknesses, all support a single conclusion.

This research used multiple data collection sources, namely teachers, learners and documents as well as different methods which were interviews, documentary analysis and observations.

3.6.2 Pilot study

Pilot testing refers to “a small scale implementation of the draft data collection instruments that assesses clarity, comprehensiveness and acceptability” (Rea & Parker, 1997, pp. 28-29). During the pilot testing, the researcher tries out the research techniques and methods which they have in mind in order to see how well they work in practice and make necessary modifications where necessary (Blaxter, Hughes & Tight, 2001). So, the data collection instruments were piloted on teachers and learners selected from three different schools within the educational zone covered in the study. The school used for the main study was not among the three schools involved in the pilot study.

By conducting this pilot testing, the researcher wanted to check if the questions were clear and also whether they would get the intended responses. During piloting, some questions got the same responses while others needed more elaboration and clarifications. As such, some questions were combined while others were revised and refined to make them clear. In addition, the researcher observed that it was tedious for learners, irrespective of grade, to add and subtract numbers that were greater than 30 due to some of the methods they were using to work out such number tasks like counting-all. Therefore, number tasks that had such bigger numbers were removed from the main data

collection instrument for learners leaving only those that were manageable and challenging for learners.

3.6.3 Direct quotations

Hancock (2002) emphasises that “quotations should be used because they are good examples of what people have said specifically about the category being described” (p. 27). In this case, quotations add strength to a researcher’s claim because they speak for themselves. In view of this, the researcher selected a range of direct quotations, from the analysed data which captured the experiences and perspectives of research participants. This was done to illustrate a claim or an opinion, similarities and width of ideas.

3.6.4 Respondent validity

The researcher made sure that at the point of data collection, research participants were not coerced to participate in the study. Rather, they were asked to get involved in the study willingly and were also well informed of their right to withdraw from the study at any time they felt it was not in their best interest. In order to accurately capture the views of the research participants especially teachers during interviews, the researcher shared his interpretation of their responses with what they meant, a process referred to as respondent validity (Silverman, 2005).

3.7 Access negotiation to the research site

In any study, access negotiation or gaining access to the research site through gatekeepers and establishing rapport with a case, is crucial (Mukherji & Albon, 2010). In order to

gain access to the study primary school, the researcher sought permission from the District Education Manager (DEM) for Balaka district in writing (Appendix A) who granted the permission (Appendix B) and referred the researcher to the study school through the Primary Education Advisor (PEA).

Prior to the beginning of the study, the researcher visited the school to make arrangements with the school administration. The researcher briefed the head teacher about the topic of the study and the type of participants to be involved in the study. Thereafter, the head teacher invited the concerned teachers (Standards 1 and 2 mathematics teachers) and together with the researcher, briefed them on the same. Participating teachers were requested to commit themselves by willingly signing a letter of consent (Appendix C) before the beginning of the study. For learners, they were asked to commit themselves verbally. Thereafter, the researcher informed and asked their parents to sign a letter of consent (Appendix D).

On the first day of the data collection process, the researcher approached the head teacher to help in making arrangements. The head teacher identified a room for conducting interviews for both teachers and learners. The researcher started with interviewing teachers for one week during which one teacher was interviewed per day. The interview sessions took 31 to 48 minutes. The durations varied due to interests shown by some respondents and probing done by the researcher during interviews. By starting with interviews, the researcher wanted to achieve familiarity with teachers before observing their lessons so that they could teach the way they always did.

Teachers' interviews were followed by lesson observations. One teacher per day was observed teaching mathematics on number concepts: counting, addition and subtraction. Mathematics syllabi for Standards 1 and 2 offer these three number concepts with Standard 1 offering single digit numbers (0 to 9) and Standard 2 offering up to two digit numbers (0 to 99). As such, mathematics teachers were already teaching these number concepts right from Term 1 of the 2015/2016 school year. So, the researcher was incorporated into the teachers' existing teaching programme. These lesson observations were followed by analyses of teachers' schemes of work, lesson plans, learners' exercise books and classroom resources for mathematics.

Learners' oral assessment interviews were administered during the last two and a half weeks. The researcher wanted learners to be familiar with the researcher during lesson observations so that they could feel comfortable during the interviews considering the nature of young children in these classes. In total, the study took five and a half weeks from 18th January, 2016 to 24 February, 2016. This period also provided room for interviewing learners who were absent on some days.

3.8 Ethical considerations

Research is a moral and ethical enterprise, and should aim at ensuring privacy and interests of research participants to avoid inflicting harm on them for taking part in the study (Human-Vogel, 2008). This is achieved by striking a balance among the pursuits of knowledge, the rights of the research participants and those of others in society (Neuman,

2003). In view of the above, ethical issues were considered at two levels namely; informed consent and confidentiality and anonymity.

3.8.1 Informed consent

Fraenkel and Wallen (2009) advise that potential research participants in a study should give their consent only after being well informed of the goals of the study and factors that could influence their decision making. In view of this, the researcher wrote a consent letter (Appendix C) for mathematics teachers in which they gave their consent by signing. This was done after explaining the contents of the letter verbally. The content included the aim of the study and how it was to be conducted. All selected mathematics teachers showed their interest to participate in the study and all signed the consent forms. The researcher also sought teachers' consent on the use of an audio and video-recorders to which they all agreed.

For the learners, the process of seeking their informed consent was not straight forward. According to Coady (2001, p. 64), children are "heavily represented among victims of research". However, though vulnerable, children are competent and have the ability to participate in research (Mukherji & Albon, 2010). Thus, both "the notions of competent and vulnerable, worded as competent yet vulnerable child, may be held simultaneously as a way of considering the unique position of children" (Lahman, 2008, p. 285). In this regard, Mukherji and Albon stress that research studies involving young children need to be treated with special attention regarding ethics owing to the age and vulnerability of young children. Robson (1993) also maintains that there are ethical problems relating to

informed consent when working with vulnerable groups such as children, especially very young children because it is difficult for them to fully understand every aspect of the research. As such, it is difficult to tell whether they “can rationally, knowingly and freely give informed consent” (Robson, 1993, p. 32). For this reason, Robson advises that researchers working with young children have to ask the children’s parents or guardians for written permission to carry out the research.

In view of the above arguments, the researcher sought informed consent at three levels. Firstly, after explaining the aim of the study to the whole class and what was expected of them, the researcher sought verbal consent from each of the selected children in the sample. Secondly, the researcher wrote a letter to each of the selected children’s parents or guardians (Appendix D) to seek informed consent for their children to participate in the study. They were asked to indicate their consent by signing the letter to which they all signed. In the letter, the researcher explained the goal of the study, informed the parents and guardians of their child’s involvement in the study, and what was expected of them. Lastly, Langston, Abbott, Lewis and Kellett (2004) advise that very young children and babies are able to give or withdraw their consent to research. They might do this in a variety of ways such as refusing to engage with the researcher, becoming abnormally quiet, turning away and crying (or sounding distressed), and refusing to engage with any materials used in the study. Therefore, the researcher was sensitive to all of these issues in order to pick up on each child’s cues.

On top of negotiating consent with the children on a moment-by-moment basis (Langston et al., 2004), the researcher worked closely with the mathematics teachers and also sought insights of each child from the mathematics class teachers, children's parents and guardians.

3.8.2 Anonymity and confidentiality

The essence of anonymity and confidentiality in research is that information obtained from research participants should in no way disclose their identities and that the information is secretly kept (Fraenkel & Wallen, 2009). To do this, Rossman and Rallis (2003) maintain that two things should happen. First is to protect the privacy -identities, names and specific roles- of the research participants. To this end, the researcher assured participants that he would never reveal or mention their names or schools anywhere and to anyone without their prior consent. This surely freed the participants to give honest and more trustworthy answers. This was indicated when one participant said; *“Apo ndiye zilibwino. Timaopatu kuti mwina ma lecturer anu akafuna kudziwa kuti anayankhula zakutizakuti ndani”* (Now it's okay. We were afraid that your lecturers would like to know who said what during the interviews you are going to conduct).

3.9 Chapter Summary

This chapter has described the design of the study, the instruments that were used to collect data, how research participants were selected and how data were collected and analysed. It has also discussed issues to do with trustworthiness of the study, access negotiation to the study site and ethical issues.

The next chapter presents and discusses the findings of the study with regard to the children's learning of number in lower classes in Malawian primary schools.

CHAPTER 4

FINDINGS AND DISCUSSION

4.0 Chapter overview

This chapter provides details of the findings of the study. However, before presenting the findings, the chapter begins by giving a general description of the school under study in terms of characteristics of teachers and learners in order to put the discussions of the major findings in context.

4.1 Characteristics of mathematics teachers

In order to understand the findings, the study gives a summary of the Standards 1 and 2 mathematics teachers in terms of their education and professional qualifications as well as their teaching experiences.

4.1.1 Age and sex of mathematics teachers

A total of 5 mathematics teachers took part in the study. Out of these teachers, 2 were female and 3 were male. In addition, 4 of the teachers (all 3 females and 1 male) were aged between 31 and 40 years while the fifth teacher (male) was more than 40 years.

4.1.2 Education and professional qualifications of mathematics teachers

All the 5 mathematics teachers who took part in the study had a Malawi School Certificate of Education (MSCE). In Malawi, an MSCE certificate is awarded to a candidate who has successfully completed four years of secondary education and has also passed the Malawi School Certificate of Education Examinations. In addition, all the 5 teachers had received initial primary teacher education training and were all qualified primary school teachers.

4.1.3 Teaching experience of mathematics teachers

Out of the 5 mathematics teachers, 1 (male) teacher was most experienced with more than 15 years teaching experience as well as mathematics teaching experience. Two (1 male and 1 female) of the remaining 4 teachers were less experienced having 5 years teaching experience. The remaining 2 (females) teachers had less than 5 years teaching experience. In terms of mathematics teaching, all the 4 teachers were less experienced, having taught mathematics for less than five years.

4.2 Characteristics of learners

4.2.1 Learners' ages and sex

The study used a sample of 12 learners selected from Standards 1 and 2 (6 learners from each standard). For purposes of confidentiality, these learners were identified as Learner 1, Learner 2, Learner 3, up to Learner 12. Learners 1 to 6 were selected from Standard 1 and Learners 7 to 12 were selected from Standard 2. Out of the 12 learners, 7 were male and 5 were female. Figure 1 summarises the ages of the learners.

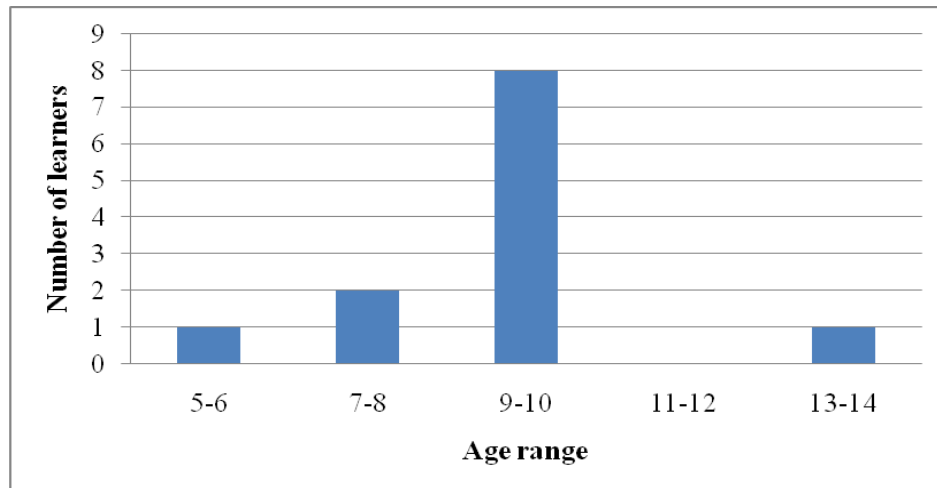


Figure 1: Learners' ages (years)

As can be seen in Figure 1 above, the ages of the learners ranged from 5 to 14 years, with 8 of them aged between 9 and 10 years. These learners were selected on the basis that they could show progress in learning number concepts as well as an average ability to speak during oral assessment interviews. As such, it was discovered during these clinical interviews that, generally, Standard 2 learners were older (10 to 13 years) than their Standard 1 counterparts (5 to 10 years).

4.2.2 Learners' background information

Information on learners' background was collected on six areas; namely whether they attended nursery/pre-school or not, language they mostly speak at home, whether someone help them with school work at home or not and whether they have reading books or magazines at home or not. They were also asked about whether or not they watch TV, and if so, where they do that, and finally whether they listen to radio or not and, if yes, where they do so. Table 8 below summarises information on learners' background.

Table 7: Learners' background information

Category	Number of learners
Language mostly spoken at home (Chichewa)	12
Got help with school work from home	12
Watched television	9
Listened to radio	9
Attended pre-school	8
Had reading books or magazines at home	0

As is indicated in Table 7, all the learners mostly speak Chichewa and also have someone helping them with school work at home. In addition, 9 of the learners in each case watch television (TV) and listen to radio at home, trading centre or elsewhere. Finally, 8 learners attended pre-school before they started formal schooling while none of the learners have reading books or magazines at home.

4.3 Results from interviews with mathematics teachers

4.3.1 Teaching methods used by mathematics teachers

Teachers were asked about methods (ways) they use to teach early number concepts (counting, addition and subtraction). They reported that they use several strategies including demonstration, question and answer, group work and think-pair-share. Table 8 below summarises what the teachers reported about methods they use to teach these number concepts.

**Table 8: Methods used by teachers to teach early number concepts
(counting, addition and subtraction)**

Teaching method	Number of teachers
Demonstration	5
Explanation	5
Group work	5
Question and answer	4
Pair work	3
Think-pair-share	1
Individual work	1

Table 8 above indicates that all 5 teachers use demonstration, explanation and group work. In addition, 4 teachers use question and answer while 3 use pair work and 1 teacher in each case uses think-pair-share and individual work. The following verbatim excerpts represent what teachers reported:

To teach counting, I have been using group work, demonstration and explanation. I have been using demonstration, explanation, question and answer, individual work, pair work and think-pair-share to teach addition of numbers. For subtraction, the methods I have been using to teach that concept are the same as in addition. [**Teacher E, 18/0116**]

... nthawi zina ana timawaika m'magulu kuti awerenge ndi kulemba manambala...Pophunzitsa kuphatikiza manambala, ndimayamba kaye kuwaonetsa ana m'mene timaphatikizira polemba m'malere. Pophunzitsa kuchotsera manambala, ndimachita chimodzimodzi ngatimomwe ndimachitira ndikamaphunzitsa kuphatikiza manambala. Ndimawaonetsa,

kuwafunsa, kuwalongosolera... (For counting, sometimes we use groups for learners to practise number counting and writing... For addition, I start teaching this concept by demonstrating how to add two numbers in the air. For subtraction, it is the same as addition. I use demonstration, question and answer, explanation...)

[Teacher C, 22/01/16]

These findings reveal that, generally, the teachers involved in this study use similar methods of teaching number concepts with demonstration, group work and explanation topping the list. This could be probably due to the fact that they use teaching methods that are suggested in their mathematics teachers' guides.

4.3.2 Learners' experiences in the teaching and learning of early number concepts

Teachers were asked about learning experiences and activities in which learners were involved in the teaching and learning of the early number concepts under study. Teachers reported that they engage learners in counting activities using physical objects such as bottle tops and sticks. Teachers also explained that learners are given experiences in number reading and writing, moulding and oral counting especially in ascending order. Table 9 below gives details of teachers' responses about learning experiences and activities that learners are involved in during the teaching of early number concepts.

Table 9: Learners’ experiences in early number concepts

Learning Experiences	Counting using objects	Counting without using objects (Verbal counting)	Reading and writing numbers	Moulding numbers
Number of teachers	5	4	1	1

Table 9 above shows that all 5 teachers engage learners in the early number concepts through use of physical objects whereas 4 of the teachers engage them orally. Finally, the table also indicates that 1 teacher in each case engages learners in number reading, writing and moulding. Some responses that teachers gave about learners’ activities and experiences are given in the following verbatim excerpt:

In these groups, learners count numbers using real objects such as ... bottle tops, sticks and learners themselves. They also mould and model numbers and count them... For 1 they count 1 object, 2 objects for 2, 3 objects for 3... To teach addition and subtraction, first we start with teaching meanings of these signs “+”, “=” and “-” saying the cross-like symbol (+) means “*phatikiza*” *kapena* “*kuphatikiza*” *kapenanso* “*kuonkhetsa*” (addition). The dash-like symbol (-) means “*chotsa*” *kapena* “*kuchotsa*”(subtraction), while the symbol “=” means “*zikhala*”(“equals” or “is equal to”). For addition like 2 + 3, together with learners we count 2 counters and put them separately, and then we count 3 others and put them separately. Thereafter, we put them together and count them all from 1.
[Teacher B, 20/01/16]

These findings reveal that generally, learning experiences and activities about early number concepts revolve around the use of physical objects as some teachers reported that the use of real objects is suitable for learners at this level of primary school.

4.3.3 Learning experiences that seemed to assist learners to grasp early number concepts and reasons

Teachers were asked about learning experiences that seem to assist learners in acquiring the concept of early number and reasons for that. Teachers' responses to this question are summarised in Table 10 below

.

Table 10: Learning experiences and activities that seemed to assist learners in acquiring early numbers concept and reasons

Learning experiences	Reason	Number of teachers
Counting using objects	Learners find it easy to do or model	3
	Learners can touch, move or remove the materials	2

As can be seen from Table 10, all 5 teachers reported that using real objects such as sticks and leaves helps learners understand early number concepts. For this, 2 teachers reported that with real objects, learners can easily model the concepts while 3 of the teachers indicated that learners can easily move or remove the objects as they count, add or subtract them. Typical views of teachers about this issue are highlighted in the following verbatim excerpts:

Of these activities, most children find it easy when I demonstrate how to count, add and subtract numbers using real objects because they observe and copy from me. **[Teacher E, 18/01/16]**

Most learners find it easy to count, add and subtract numbers when we use modeling... [Teacher D, 22/01/16]

4.3.4 Children's strategies in early number concepts

Teachers were asked about strategies learners use to acquire number counting, addition and subtraction. All 5 teachers reported that learners use real objects or verbal counting to solve number problems involving these number concepts. In addition, 2 of the teachers reported that some learners do not use any physical objects at all. Instead, they just come up with answers to some number problems straight away. The following responses are typical of what teachers reported:

To work out counting, addition and subtraction problems, learners use their fingers. [Teacher A, 19/01/16]

... Komabe ena amangofikira kutchula ansala. Amenewa amakhala kuti mwina abwereza kangapo....ukamadutsanso amakhala kuti concept ya nambala ija idakhazikika kale m'mitu mwawo... (... However, some can arrive at the correct answer quickly without using counters or fingers. These children may have repeated the same grade several times and have a well developed number concept...). [Teacher C, 22/01/16]

4.3.5 Learners' misconceptions and errors in early number concepts

Teachers were asked about learners' misconceptions and errors in early number concepts. Teachers reported several of these misconceptions and errors about early number concepts and are summarised in Table 11.

Table 11: Learners' misconceptions and errors in early numeracy

Misconception or Error	Number of teachers
Inverting numbers	4
Writing wrong or failing to write number symbols	3
Writing number problems without answers (after adding and subtracting)	3
Adding instead of subtracting and vice versa	3
Skipping numbers	2
Wrong presentation of answers	2
Place value problems involving tens and ones	2
Mismatching number words and objects while counting	2
Failing to identify number symbols	1
Failing to know cardinality after counting	1

Table 11 shows that 4 of the teachers reported that learners invert numbers while 3 of the teachers in each case reported that learners write wrong or fail to write number symbols, hand in work for marking without answers, and confuse operations of addition and subtraction. In addition, 2 of the teachers in each case reported that learners skip numbers; present answers incorrectly, have problems about place value involving tens and ones, and mismatch number words and objects. Finally, 1 teacher in each case reported that learners fail to identify number symbols and find cardinality of objects.

Typical responses on learners' misconceptions and errors are given below:

.... when writing.....numbers like 6, 4, and 3... They invert these numbers. **[Teacher B, 18/01/16]**

In counting, some learners fail to identify the symbols for those numbers although they are able to count them in oral counting. **[Teacher E, 18/01/16]**

Others confuse plus (+) sign for minus (–) and vice versa. They add instead of subtract or they subtract instead of add. **[Teacher A, 19/01/16]**

Some learners have problems with place value. For example, $21 + 23$, they add 2 from the 21 and 3 from the 23 and then the remaining 1 and 2 getting 35 instead of 44. Sometimes, they add 1 from 21 to 2 from 23 especially for horizontally sums. Others start adding from left to right instead of right to left and have also problems of carrying over numbers. **[Teacher D, 21/01/16].**

While there are several misconceptions and errors, Table 11 above reveals that the most common ones are inverting numbers, writing incorrect number symbols and adding instead of subtracting and vice versa.

4.3.6 Learners' own strategies in learning number concepts

Teachers were asked if, in the course of teaching the number concepts under study, they notice children using their own strategies in learning number concepts. The results were that 2 out of the 5 teachers reported that some learners use their own strategies in dealing with certain number problems. One of these two teachers reported that such learners might have repeated same grade several times and have, therefore, become used to working out such number problems. The other 3 teachers, on the other hand, explained that learners always copied teachers' strategies used in mathematics lessons. Views of teachers about children's strategies in early number concepts are given below:

The learners learn these concepts by imitating what the teacher is doing and they don't use different methods apart from those introduced by the teacher because at their age they cannot do that; they are too young. **[Teacher E, 18/01/16]**

Eehe! Alipo ena ndithu ukangonena kuti chakutichakuti, atchuliratu kuja. Alimo ena ndithu amatha kupanga zimenezozo eehe. Pamene ukungonena kuti 1 kuphatikiza ndi 1? Atchuliratu kuti "2", eehe kusonyeza kuti experience alinayo ndithu mwina abwereza kangapo kusonyeza kuti azizolowera mwina pang'ono. (Yes, there are some learners who just instantly give you the correct answer to some problems you orally ask them. For example, when you ask "How much is 1 and 1 altogether?" they quickly give the answer "2". Yes, these learners have gained some experience because of repeating same class several times). **[Teacher C, 22/02/16]**

Children in Standard 2 don't use other methods to solve problems... apart from counters or fingers... but Standard 3, 4 onwards they do not use counters or fingers... **[Teacher D, 21/01/16]**

There is danger in thinking that learners at this level are too young to invent their own strategies of working with number concepts. In addition, thinking that learners use their own strategies as a result of repeating a particular grade several times is also unhealthy. With these in mind, mathematics teachers can end up undermining learners' abilities and failing to adequately assist them.

4.3.7 Learners' understanding of early number concepts

Teachers were asked about learners' understanding of counting, addition and subtraction. The findings were that 4 of the teachers reported that learners understand counting as number of things in a group, addition as mixing objects and finding their number, and subtraction as number of objects that remains in a group after removing some or all from

the group. The fifth teacher's responses were not clear. The responses given below are typical of what teachers reported.

Learners understand counting as “How many”, addition as “Combining and counting all of them” and subtraction as take away things from a group of things. **[Teacher A, 19/01/16]**

For learners, counting means find how many objects in a group... Addition... is combining a group of objects to another group of objects and find how many there are altogether... For subtraction, children understand as taking some or all objects from a group of objects. Example, $9 - 5$ to mean 9 objects taking away 5 objects and find how many objects remain. Learners do this operation by firstly counting 9 things, and then from the 9 count 5 things, removing them from the group and counting those remaining. Learners easily understand the meanings of these concepts because we teach them in their vernacular languages. **[Teacher E, 18/01/16]**

The teachers' responses revolve around the issue of “how many” in terms of the number of objects in a collection which learners acquire as a result of modelling.

4.3.8 Teachers' challenges in teaching early number concepts

Teachers were asked about challenges they encounter in the course of teaching of early number concepts. Table 12 given below summarises what teachers reported about their challenges.

Table 12: Challenges teachers face in the teaching of early number concepts

Challenges	Number of teachers
Learners' lack of background knowledge in number operations	5
Absenteeism of learners	3
Large classes	2
Learners' laziness or playfulness	2
Too much curriculum content	1
Learners' loss of interest in mathematics	1
Few school learning hours per day	1

As is observed in Table 12 above, all 5 teachers reported that learners lack background knowledge of number operations while 3 complained of absenteeism of learners. Issues such as large classes and overloaded curriculum were reported by 2 of the 5 teachers. Finally, the least reported challenges include few school learning hours per day and loss of interest in mathematics by learners. Given below are some of the typical responses from teachers.

... then some learners lack basics of number operations. Again, large classes; with 200 learners that I have in my small classroom, I fail to assist learners because I cannot move about. Again, Malawian school calendar is not good because learners spend more time at home than in school despite having a congested curriculum. In other countries I hear that learners come to school at 6:00am and knock off at 6:00pm. In this case, not much is done in education in Malawi because they give us tough time since it is too short. **[Teacher E, 18/01/16]**

I face so many challenges. These include large classes. Since most learners are underage, they don't get what I am teaching and I fail to assist them. Most learners also do not come to school regularly...
[Teacher A, 19/01/16]

4.3.9 Children's challenges in early number concepts

On top of teachers' challenges, the study sought to explore challenges which prevented learners from acquiring early number concepts. Table 13 gives a summary of these results.

Table 13: Learners' challenges in counting, addition and subtraction

Challenges	Number of teachers
Absenteeism	4
Large class size	2
Inadequate teaching and learning resources	2
Lack knowledge of number operations	2
Laziness or playfulness	1
Poor teaching methods	1
Underage	1
Slow learners	1

As is highlighted in Table 13, 4 out of the 5 teachers reported that learners are frequently absent from school whereas 2 teachers in each case mentioned large class sizes, inadequate teaching and learning resources and lack of background knowledge in number

operations on the part of learners. Finally, 1 teacher in each case reported that learners are slow, playful and/or lazy, underage, and that some teachers use poor teaching methods.

The following responses were typical of what teachers reported:

... another problem is absenteeism. Most children come to school today and tomorrow they don't come. So, they miss a lot and fail to connect what their friends learnt on the days they were absent. In addition, the class is large and within 35 minute periods, it is not possible to help individual learners who are facing problems. [**Teacher B, 22/01/16**]

... most children begin school while they are under age because parents do not understand why children should begin school in good time. Most parents think that because their children went to nursery school and that the nursery school caregivers certified the children to begin Standard 1 then they can start Standard 1 [**Teacher A, 19/01/16**]

Some teachers do not use appropriate methods that can help children learn these concepts. This is because in some colleges, teacher learners [meaning "student teachers"] learn by rumours because of understaffing of college tutors or incompetent college tutors. Some tutors do not teach some of these concepts but they just say go and research in the library. So some teachers are half baked. [**Teacher E, 18/01/16**]

4.3.10 Ways teachers use to address challenges faced

Finally, teachers were asked about how they deal with the challenges they face as they teach early number concepts and those hindering their learners from acquiring the same.

Table 14 gives a summary of how the teachers overcome such challenges.

Table 14: Ways teachers deal with challenges involving early number concepts

How teachers deal with challenges faced	Number of teachers
Discuss with parents/ guardians	2
Assign work to groups not individual learners	2
Use able learners as both group leaders and instructors	2
Give homework	2
Improvise teaching and learning resources	2
Provide individual assistance	2
Firm but fair classroom management	1
Vary teaching strategies	1
Use remedial (free-extra) lessons	1
Give extra-easy work (from already covered work)	1

As revealed by Table 14, 2 out of 5 teachers in each case give learners homework, provide them with individual assistance, improvise teaching and learning resources, discuss with parents and guardians, teach learners in groups and use able learners to lead and teach fellow learners in groups as ways of overcoming challenges the challenges faced. Further to that, 1 teacher in each case tightens classroom management strategies such as spelling out dos and don'ts, uses a variety of teaching strategies, remedial classes and extra-easy work. Given below are typical examples of teachers' solutions to the challenges faced.

To solve these challenges, I provide individual assistance to learners...give homework and extra work. This extra or homework I take from previous work I already taught.... The work is normally simpler than the normal class work because I want to motivate them... For frequent absenteeism, I invite parents for discussions. **[Teacher D, 21/01/16].**

I use remedial classes... Sometimes when I am teaching, I frighten them saying those who will not get problems correct will not go home.... This makes troublemakers to concentrate while I am teaching and they do well. **[Teacher B, 19/01/15]**

These findings reveal that teachers use several solutions to the challenges they meet as they teach early number concepts. However, some of the solutions such as provision of individual assistance to learners and remedial classes are too ambitious and not practical in the face of large classes for example.

4.4 Results from learners' oral assessment interviews

Learners were given ten tasks namely; oral counting, one-to-one correspondence counting, numeral identification, addition and subtraction word problems, addition and subtraction problems, forward and backward number word sequences, number word after and number word before. Findings on each of these tasks are given in the sections that follow.

4.4.1 Results from Task 1: Oral counting

This task begins with the number 1, and asks learners to continue counting until they reach the highest number they could before making a counting error. The child's score on this task is based on the last correct number the child said before making a counting error or at the end of a minute.

The task is timed, because its purpose is to elicit a fluency measure from the learner. The results from this assessment task are presented in Figure 2 below.

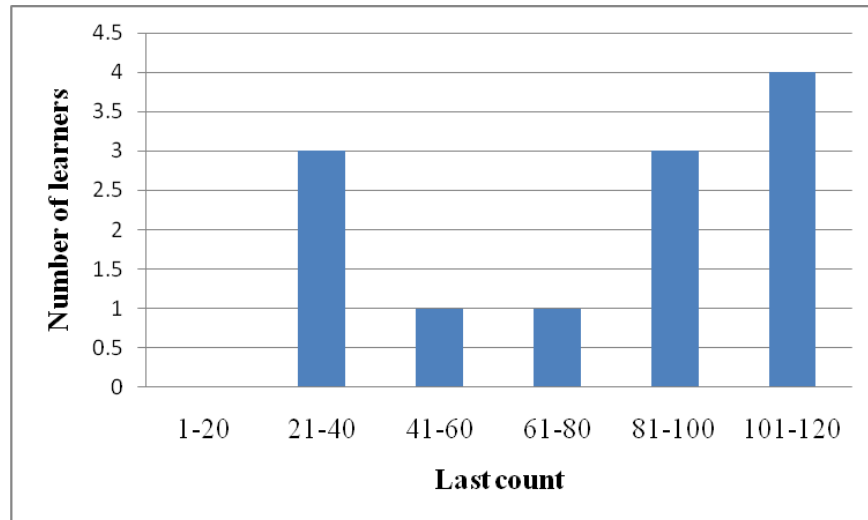


Figure 2: Results of oral counting task

Figure 2 above shows that 4 out of the 12 learners correctly counted in the range of 101 to 120. In addition, 3 of the learners in each case counted in the ranges of 21 to 40 and 81 to 100 and, finally, 1 learner in each case counted in the ranges of 41 to 60 and 61 to 80. Oral counting task provide teachers to learn children's knowledge of number names (Ginsburg & Russell, 1981). Given below are some of the learners' responses.

1, 2, 3... 109, 200 **[Learner 9, 18/02/16]**

1, 2, 3... 39, 50, 51, 52... 59, 30, 31, 32... 39. **[Learner 2, 23/02/16]**

1, 2, 3... 120. **[Learner 2, 18/02/16]**

4.4.2 Results from Task 2: One-to-one correspondence counting

Learners were given a sheet of paper containing 120 circles and then asked to count them one by one. This task was timed and given a maximum of 1 minute. The time taken by each learner to do the counting, and the last spoken correct number of circles counted were recorded. In addition, each learner was asked to say how many circles they had counted and this was recorded as well. As a child recognises each item, they tag it mentally as needing to be counted, and tagging can be done physically by pointing to the item to keep track of those still needing to be counted, as well as those that have already been counted (Gelman & Gallistel, 1986). The findings were that some learners were able to count the circles and correctly tell how many they have counted. However, others were able to count a given number of circles but were not able to indicate how many counted. Figure 3 gives a summary of results from this task.

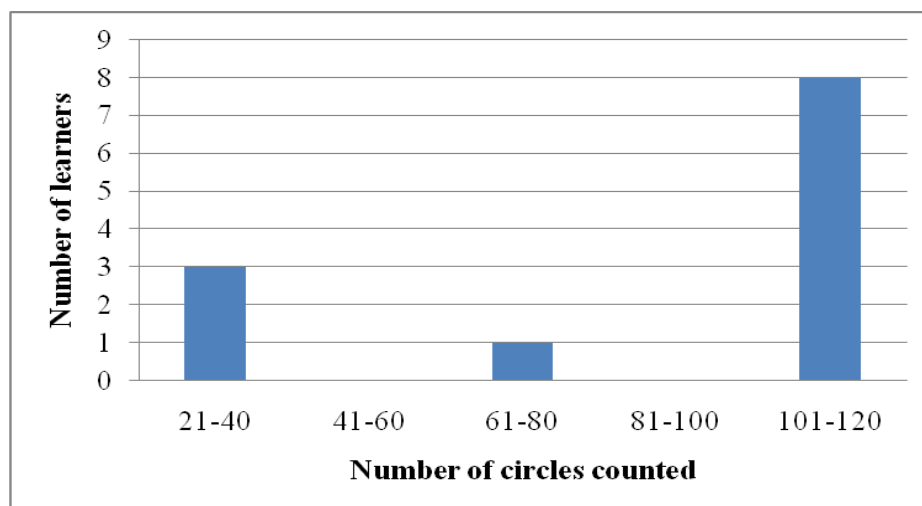


Figure 3: Results of one-to-one correspondence counting

Figure 3 reveals that 8 out of 12 learners counted between 101 and 120 circles. Each one of them correctly indicated the number of circles they had counted. Also, 3 of the learners made counted between 21 and 40. However, none of them stated correctly how many circles counted. They mentioned either less or more than the actual number of circles counted. For example, Learner 2 [Standard 1A, 23/02/16] counted 39 circles correctly but said there were 6 circles altogether. Finally, the table also shows that 1 learner counted between 61 and 80. However, the learner stated a smaller number of circles (60) than how many actually counted (62). Learners' typical responses to this task are given below.

1, 2, 3... 118. *Ma seko onse alipo 118* (There are 118 circles altogether).
[Learner 4, 24/02/16]

1, 2, 3... 39, 50, 52, 53...59, 40, 41, 42... 50. *Ma seko alipo 39* (There are 39 circles altogether). **[Learner 6, 23/02/16]**

1, 2, 3... 39. *Maseko onse alipo 6* (There are circles altogether).
[Learner 2, 23/02/16]

This task sought to find out learners' "rhythmic coordination of the partitioning and tagging process" (Gelman & Gallistel, 1986, p. 78) or simply "learners' ability in counting objects" (Reubens, 2009, p. 17). Children needed to use two processes that needed to work together, firstly by recognising items that were yet to be counted, and secondly recognising, and mentally tagging those that had already been counted (Gelman & Gallistel, 1986). Figure 4 shows how learners did this task with the aid of fingers to help them accomplish the tagging process.



Figure 4: One-to-one correspondence counting in progress showing tagging process

4.4.3 Results from Task 3: Number identification

4.4.3.1 Results from Task 3A: Number identification-exercise one

Number identification tasks seek to check an understanding of children's knowledge and identification of written number symbols (Reubens, 2009). In this study, the numbers were purposefully selected and presented in a grid to which learners were supposed to identify in printed form. The task was timed and each learner was given 30 seconds to identify 12 numbers. Figure 5 gives a summary of results from this task.

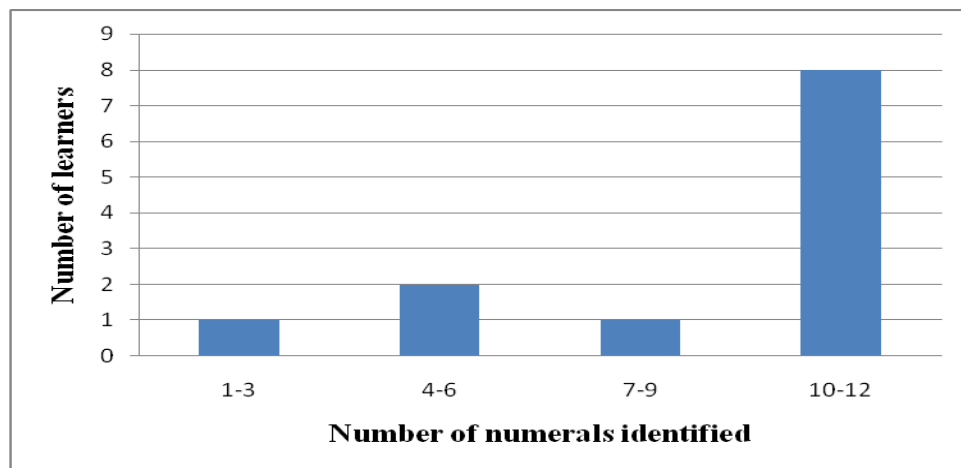


Figure 5: Results from number identification exercise one

As is revealed in Figure 5, 8 out of the 12 learners correctly identified 10 to 12 numerals whereas 2 of the learners identified 4 to 6 numerals correctly. Finally, 1 learner in each case correctly identified 1 to 3 numerals and 7 to 9 numerals. One role from the task had these numbers: 15 12 7 13. Learners' typical responses were as follows:

5	2	7	3	[Learner 2, 23/02/16]
15	12	7	13	[Learner 8, 19/02/16]

These results show that while some learners were able to identify the numbers, others, like Learner 2, had problems identifying 2-digit numbers. In addition, some learners were confusing numbers such as 12 and 21, 15 and 51, 37 and 73, and 66 and 99.

4.4.3.2 Results from Task 3B: Number identification-exercise two

As was the case with number identification Task 3A-exercise one, number identification Task 3B-exercise two sought to check an understanding of children's knowledge and identification of written number symbols (Reubens, 2009). Similarly, the numbers were purposefully selected and presented in a grid which learners were supposed to identify in printed form (Clarke & Shinn, 2004). However, number identification Task 3B differed from that of Task 3A in that it had 20 numbers and was also a bit harder. Again, each learner had 30 seconds to do the task. The results from this task are given in Figure 6 below.

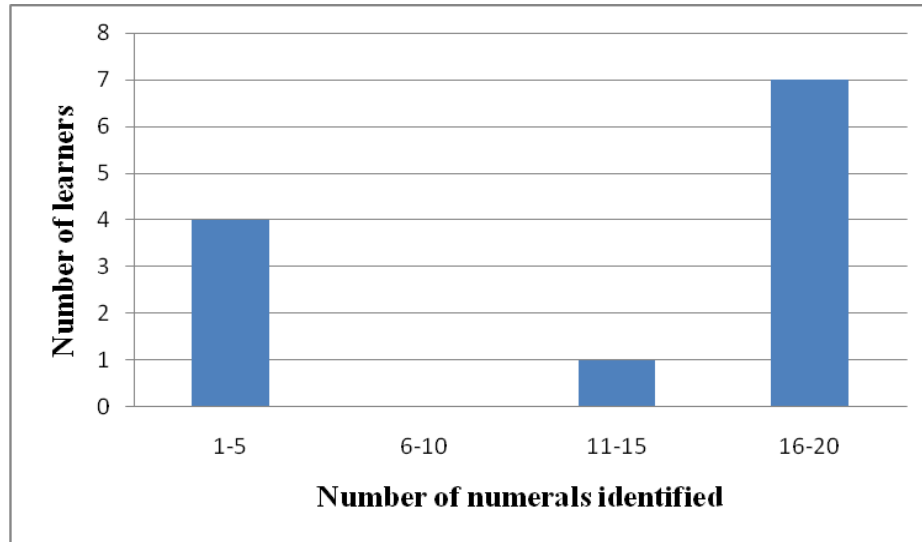


Figure 6: Results from number identification exercise two

Figure 6 above indicates that 7 of the learners correctly identified 16 to 20 numerals. Further to that, 4 of the learners correctly identified 1 to 5 numerals while 1 learner correctly identified one numeral. One row from the task had the following numbers: 73

85 99 36. Some learners' responses to this row were as follows:

73	85	99	36.	[Learner 9, 18/02/16]
37	58	99	36.	[Learner 4, 24/02/16]

As shown in the excerpts, while some learners were able to identify two-digit numbers, others were not. This partly could be due to the fact that mathematics syllabi for Standards 1 and 2 are different. Standard 1 syllabus deals with single-digit numbers only whereas Standard 2 extends up to 99. Despite this limitation for Standard 1 children, they were able to identify numbers far beyond the scope of their syllabus.

4.4.4 Results from Task 4: Addition and subtraction word problems

This task had 4 number word problems. Learners were provided with counters, a paper and a pencil that they could use in solving the problems. The instruction required that the researcher reads the entire word problem to a child before they could begin the task (Reubens, 2009). If the child needed a word problem re-read, the researcher re-read it in its entirety. Each problem was re-read as often as the child needed, as doing so helped him or her continue with the identification of the numbers while solving the problems. In the process of child's solution, the researcher took note of or inquired about children's strategy on each number word problem. Results from this task are presented in Table 15 below.

Table 15: Learners' strategies in addition and subtraction word problems

Strategy	Number of Learners
Mastery/Automatic	6
Count-all	4
Count-on	1
Guessing	1
Total	12

Table 15 above shows that 6 out of 12 learners found solutions without using any counting aid as they gave answers straight away. So, they used mastery or automatic strategy. Further to that, 4 of the learners used count-all whereas 1 learner in each case used count-on strategies and guessing. With count-all strategy, a child finds the total

number of objects in a collection by counting them from 1. For example, for $3 + 2 = \underline{\quad}$, a child will count 3 objects (“1, 2, 3”), put them aside; count 2 objects (“1, 2”) and put them aside as well. Thereafter, he or she will put these objects together and count them from 1 (“1, 2, 3, 4, 5”) to get 5. A child using a count-on strategy will keep either number in mind and continue counting by ones with the other number. Using the $3 + 2 = \underline{\quad}$ above, the child will start from either “3” and make 2 counts saying “4, 5” or will start from “2” and make 3 counts saying “3, 4, 5” and get “5” in either case. The child can make these counts verbally or use fingers or other objects to help keep track.

One question from this task was: *Jane anali ndi mabisiketi 4. Bambo ake a Jane adamupatsa mabisikti ena awiri. Kodi Jane anali ndi mabisiketi angati onse pamodzi?* (Jane had 4 biscuits. Her father gave her 2 more biscuits. How many biscuits does Jane have altogether?) Some learners’ responses were as follows:

(Instantly) “6”. **[Learner 10, 22/02/16] (Mastery strategy)**

(Counts while bending 2 fingers saying) “5, 6”. “6”. **[Learner 3, 23/02/16] (Count-on strategy)**

(Counts while bending fingers saying) “1, 2, 3, 4”. *Kenaka* (Then) “1, 2”. *Zonse pamodzi* (altogether) “1, 2, 3... 6.” “6”. **[Learner 9, 18/02/16] (Count-all strategy)**

These findings agree with what teachers pointed out during interviews that learners use real objects such as bottle tops and fingers to help them count, add and subtract numbers. However, as opposed to what teachers reported, 6 learners used automatic strategies. Therefore, the use of multiple data collection instruments assisted in capturing additional strategies which were not highlighted by teachers.

Learners' strategies in addition and subtraction word problems were also compared in relation to the ages of the learners. Out of the 12 learners, 1 learner in the age range 5 to 6 used count-all strategy whereas another one in the age range 7 to 8 used mastery. For the age range 9 to 10, 1 learner in each case used guessing and count-on, 4 learners used count-all, and 3 others used mastery strategies. Further details on learners' strategies in of word problems in relation to their ages are summarised in Table 16.

Table 16: Learners' ages versus types of strategies in addition and subtraction word problems

Learners' Ages	Guessing	Count-all	Count-on	Mastery	Total
5 to 6		1			1
7 to 8			1	1	2
9 to 10	1	2	1	4	8
11 to 12					
13 to 14				1	1
Total	1	3	2	6	12

Table 16 shows that 4 learners, aged 5 to 10, used guessing and count-all while 8 out of 12 learners aged 7 years and above used count-on or mastery, with more learners moving away from blindly guessing and use of counting strategies towards automatic recall of answers for small numbers for older learners. This trend indicates that learners' abilities regarding the types of strategies used generally depend on their ages.

4.4.5 Results from Task 5: Addition and subtraction problems

In these tasks, children were shown a visual representation of the mathematics problem, and also had the problem read aloud to them. In addition, they were provided with counters and a paper and a pencil so that they could use if necessary. Children could use any strategy in solving the problems and the researcher observed or inquired about each child's strategy on each number problem.

4.4.5.1 Results from Task 5A: Addition problems

This task had a total of six number problems. Learners were given one problem to work out at a time. Learners used a variety of strategies to work out the addition number problems which are summarised in Table 17 shown below.

Table 17: Learners' strategies in addition problems

Strategy	Number of Learners
Count-all	5
Count-on	2
Decomposition	3
Mastery/Automatic	1
Guessing	1
Total	12

Table 17 reveals that 5 out of the 12 learners used count-all, 2 learners worked via count-on, 3 others used decomposition strategies (number splitting into constituent parts, facts or relationships to make a problem easier to work out) while 1 learner in each case used

mastery and guessing. Figure 7 shows strategies that learners used to deal with the number tasks:



(Count-on or count-all strategy)

Figure 7: Addition problems in progress

For the number task: $8 + 7 = \underline{\quad}$: 8 kuphatikiza 7 yankho lake ndichiyani? [How much is 8 and 7 altogether?]

(Counts some numbers silently while bending fingers). “15”. *Ndinatere. Ndinati 8, kenako 9, 10, 11... 15.* [“15” I did this. I said 8, then 9, 10, 11...15] **[Learner 1, 18/02/16]. (Count-on strategy)**

For the number problem: $9 + 5 = \underline{\quad}$: 9 kuphatikiza 5 yankho lake ndi chiyani? (What is 9 plus 5?)

Ujeni, ujeni, “14”. Ndinatenga 1 ku 5 nkuphatikiza ku 9 uja kukhala ngati 10. Ndiye kenako 10 ndi 4 uja anatsalayu zinakhala ngati 14. (That is, that is, 14. I subtracted 1 from 5 and added it to 9 making 10. Then 10 together with the 4, which remained from the 5, equalled 14). **[Learner 8, 19/02/16](Decomposition strategy)**

For the number problem: $12 + 13 = \underline{\hspace{1cm}}$: *12 kuphatikiza 13 yankho lake ndi chiyani?*

[How much is 12 and 13 altogether?]

(Blinks for 4 seconds). “25”. *Ndinatere: 13 kuchotsa 10 atsala 3; 12 kuchotsa 10 atsala 2. 10 kuphatikiza 10 tipeza 20, 2 kuphatikiza 3 tipeza 5. Ndiye 20 uja kuphatikiza 5 yankho lake ndi 25. Nchifukwa chake ndinati “25”. (“25”. 13 minus 10 leaves 3 while 12 minus 10 leaves 2. 10 plus 10 is 20 and 3 plus 2 gives 5. Finally, 20 plus 5 is 25. That is why I said 25).* [**Learner 7, 23/02/16**] (**Decomposition strategy**)

Thus, while some learners used real objects as their teachers rightly indicated during interviews, others invented their own strategies such as decomposition, compensation and mastery (or retrieval from memory especially for small numbers like $3 + 2 = \underline{\hspace{1cm}}$). In addition, some learners found answers to some number problems by guessing blindly. It has to be pointed out here that (blind) guessing appears on the continuum of strategies in Table 17 above purely on the basis of its frequency. It does not represent any level of sophistication in terms of children’s strategy as it uses chance not reasoning especially at this level.

Learners’ strategies on addition number problems were also compared in relation to their ages. The results were that 1 learner in the age range 5 to 6 and 2 learners aged between 7 and 8 used count-all strategy. In the age range 9 to 10, 3 learners in each case used count-on and decomposition strategies whereas 2 used mastery. Finally, 1 learner in the age range 13 to 14 used decomposition strategies. These findings are summarised in Table 18 below.

Table 18: Learners' ages versus types of strategies used in addition problems

Learners' Ages	Count-all	Count-on	Decomposition	Mastery	Total
5 to 6	1				1
7 to 8	2				2
9 to 10		3	3	2	8
11 to 12					
13 to 14			1		1
Total	3	3	4	2	12

Table 18 reveals that out of 12 learners, 3 learners aged between 5 and 8 years used count-all strategy while the rest (aged 8 years and above) tended to use count-on, decomposition and mastery strategies. This shows that learners' ages had a bearing on learners' abilities regarding the strategies used. In particular, older learners generally tended to use more of decomposition and mastery strategies than those involving counting as did younger children.

4.5.1.2 Results from Task 5B: Subtraction problems

Learners were also assessed in subtraction problems. As was the case with the addition problem task, this sub task had 6 questions and learners were given one problem at a time to solve. Table 19 gives a summary of learners' strategies in subtraction tasks.

Table 19: Learners' strategies in subtraction problems

Strategy	Number of learners
Count-all	9
Mastery/Automatic	1
Decomposition	2
Total	12

As is indicated in Table 19 above, 9 out of 12 learners approached the problems using count-all strategy. For the remaining 3 learners, 1 used mastery while the other 2 used decomposition (known number relationships or facts) strategies. As compared to addition problems, these findings reveal that, generally, learners found subtraction tasks more challenging as more (9) learners relied on counting-by-ones strategies like count-all. The following oral assessment interview extracts demonstrate some of the strategies that were noted among learners in subtraction problems: For the task: $15 - 4 = \underline{\quad}$: *15 kuchotsa 4 yankho lake ndi chiyani?* (How much is 15 take away 4?).

(Counts 15 objects, counts 4 things from the 15 and puts them aside. Then counts the remaining counters to get 11) “11”. **[Learner 12, 19/02/16].**
(Count-all strategy).

For the task: $23 - 7 = \underline{\quad}$: *23 kuchotsa 7 yankho lake ndi chiyani?* (How much is 23 take away 7?).

(Puts hands and feet together) “13”. “*Ayi, 16*”. *Ndinaphatikiza kuti 21, 22, 23 chamumtima. Kenako ndinati 13; kenaka 14, 15, 16.* (“13”. “No, 16”. I counted silently saying 21, 22, 23. Then from 13, I counted 14, 15, 16). **[Learner 1, 18/02/16]. (Counting-all- combined with counting-on strategies)**

For the task: $7 - 4 = \underline{\quad}$: *7 kuchotsa 4 yankho lake ndi chiyani?* (How much is 7 take away 4?)

(2 seconds) “3”. *Ndinati 8 kuchotsa 4 ndi 4. Ndiye 7 kuchotsa 4 ndi 3.* (I said since 8 take away 4 leaves 4, then 7 take away 4 is 3).

[Learner 1, 24/02/16]. (Compensation strategy or decomposition using knowledge of doubles)

For the numerical problem: $9 - 6 = \underline{\quad}$: *9 kuchotsa 6 yankho lake ndi chiyani?* [How much is 9 take away 6?]

(Quickly) “3”. *Adakakhala 10 n’dakapeza 4, koma ndi 9 kusonyeza ngati kuti pa 10 paja tachotsa 1. Ndiye ansala ikhala 4 kuchotsa 1 imene ndi 3.* (If it were 10, I would find 4; but it is 9. It is as if we have taken away 1 from 10. So the answer is 4 minus 1 which is 3).

[Learner 8, 19/02/26] (Decomposition or compensation strategy)

These findings show that at some point, learners combine strategies to complement one another just like what Learner 1 above did. Learners can invent strategies when they have acquired fluency of working with numbers of their range.

As was the case with addition and subtraction word problems, and addition problems, learners’ strategies for subtraction problems were compared in relation to their ages. The findings were that 1 learner in each case in the age ranges of 5 to 6 and 13 to 14, 2 learners in the age of range 7 to 8, and 5 learners in the age range of 9 to 10, all used count-all strategy. The remaining 3 learners were in the age range of 9 to 10 and all used decomposition strategies. Table 20 below gives a summary of these findings.

Table 20: Learners' ages versus their strategies in subtraction problems

Learners' Ages	Count-all	Decomposition	Mastery	Total
5 to 6	1			1
7 to 8	2			2
9 to 10	6	2		8
11 to 12				
13 to 14			1	1
Total	9	2	1	12

Table 20 shows that 9 learners aged 5 to 10 years used count-all strategy where as 3 learners aged 9 to 14 years used decomposition and mastery strategies. Although children generally found subtraction problems challenging, few older children tended to use non count-by-ones strategies, still indicating that the ages of learners determined their strategies in subtraction problems.

4.5.2 Results from Task 6: Number identification

The task was aimed at exploring learners' ability to recognise and identify numerals that were purposively selected and arranged in a grid as was the case in Task 3. However, Task 6 (as opposed to Task 3) was also aimed at determining levels that learners had reached in numeral identification, using the model for numerical identification (Table 3 on page 67) given in Chapter 2. Learners were given numbers to identify in four

categories namely; emergent numerals to 10, numerals to 10, numerals to 20 and numerals to 100. The category for emergent numerals to 10 was to detect learners who could not correctly identify all given numbers in the range 1 to 10 because they either did not know some number words or that they could not match number words and symbols (Wright, Martland & Stafford, 2000; 2006). Table 21 gives a summary of results from this task.

Table 21: Children's abilities in numeral identification (NI)

Name of Category	Number of Learners
Emergent Numerals to 10	2
Numerals to 10	4
Numerals to 20	1
Numerals to 100	5
Total	12

The table reveals that 5 out of 12 learners identified numerals to 100 while 2 learners were yet to identify numerals to 10. In addition, 4 learners identified numerals to 10 whereas 1 identified numerals to 20. At the time of this study, Standard 1 learners had been introduced to all numbers of their range (1 to 9) as per their mathematics syllabus while those in Standard 2 had been introduced to numbers from 1 to 70 and were yet to be introduced to those from 71 to 99 as per their syllabus also. Despite these differences, learners from the two classes were able to identify numbers up to 100. This could be because of the informal mathematics they meet in everyday life. One category, numerals

to 100, had these numbers: 23 77 56 100 98 20. Given below are typical responses from the learners.

23 77 56 100 98 20. [Learner 12, 19/02/16]

23 77 56 100 98 20. [Learner 3, 23/02/16]

32 77 65 100 86 20. [Learner 7, 23/02/16]

These findings reveal that some learners were conversant with numerals to 100 such as Learner 3 and Learner 12 while others like Learner 7 had problems indentifying some numbers such as 32 for 23 and 86 for 98.

4.5.3 Results from Task 7: Number word sequences

4.5.3.1 Results from Task 7A: Forward number word sequence (FNWS)

FNWSs tasks were aimed at exploring children's ability to say FNWSs and later determine each child's level using the FNWS model (Table 4 on pages 68-69) as already discussed in Chapter 2. The numbers were in six categories and one such category was "facile" FNWSs up to 100. This category had two sub categories, and one of which was 49 to (67). Learners were only told where to start from (49) and not where to finish (67). The researcher had to stop the learner upon reaching the number in bracket of each category. Table 20 gives a summary of results from this task.

Table 22: Learners' abilities in forward number word sequences (FNWSs)

Name of Category	Number of Learners
Emergent FNWS to 10	0
Initial FNWS up to 10	1
Intermediate FNWS up to 10	1
Facile with FNWS up to 10	1
Facile with FNWS up to 30	3
Facile with FNWSs up to 100	6
Total	12

Table 10 reveals that 6 of the 12 learners flexibly made FNWSs up to 100. It also shows that 3 other learners were flexible with FNWSs up to 30. Except for the category of emergent FNWSs which had no learners, the remaining categories had one learner each. Given below are FNWSs said by learners for the category of facile FNWSs up to 100, sub category 49 to (67).

(Quickly) 49, 50, 51... 67. **[Learner 8, 19/02/16]**

(Counts loudly while bending fingers followed by toes) 49, 50, 51... 59, 30, 31, 32. **[Learner 6, 23/02/16]**

49, 42, 43, 44, 45... 48, 90, 91, 92, 93, 94. **[Learner 5, 23/02/16]**

4.5.3.2 Results from Task 7B: Backward number word sequences (BNWSs)

The study sought to explore children's abilities in saying BNWSs and then determine their levels in BNWSs using a model for BNWS (Table 5 on pages 70-71). Numbers in this task were put into six categories as was the case with FNWSs. The findings from this task are summarised in Table 23 below.

Table 23: Learners' abilities in backward number word sequences (BNWSs)

Name of Stage	Number of Learners
Emergent BNWS to 10	2
Initial BNWS up to 10	1
Intermediate BNWS up to 10	2
Facile with BNWS up to 10	2
Facile with BNWS up to 30	5
Facile with BNWSs up to 100	0
Total	12

Table 23 indicates that none of the learners was facile with BNWSs up to 100. In addition, 5 learners were flexible saying BNWSs up to 30, 2 learners in each case were facile with BNWSs up to 10 and intermediate BNWSs to 10 while 1 learner was in the initial BNWSs up to 10. Finally, 2 learners were in the emergent stage of BNWSs to 10. One category was "facile" with BNWSs from 1 to 100 and was further divided into two sub categories 83 to (74) and 61 to (52). For the 83 to (74) sub category, some learners' responses were as follows:

83	82	81	80	90	99	98	[Learner 12, 19/02/16]
83	82	81	70	71			[Learner 7, 23/02/16]

As can be observed from the responses, learners found BNWSs a bit challenging as compared to FNWSs.

4.5.4 Results from Task 8: Number word after and number word before

The study sought to explore children's abilities to mention number word after (NWA) and number word before (NWB) given a list of numbers that were purposively selected. Thereafter, levels which learners had reached in NWA and NWB were determined using models for FNWSs (Table 4 on pages 68-69) and that for BNWSs (Table 5 on pages 70-71) respectively due to their relatedness.

4.5.4.1 Results Task 8A: Number word after (NWA)

Numbers in this task were in six categories. The results were that 7 out of 12 learners were facile in NWA up to 100 while 1 learner was still in the emergent NWA to 10. Details of these results are summarised in Table 24.

Table 24: Results of learners' abilities in number word after (NWA)

Name of Stage	Number of Learners
Emergent FNWSs to 10	1
Initial FNWSs up to 10	2
Intermediate FNWSs up to 10	1
Facile with FNWSs up to 10	1
Facile with FNWSs up to 30	0
Facile with FNWSs up to 100	7
Total	12

Table 24 reveals that, as was the case with FNWSs, learners found NWA task generally easy, with 7 out of 12 learners in the facile NWA to 100, and only 1 learner in the emergent NWA to 10. The remaining 4 learners were in between these two extremes.

One category, facile with NWA up to 30 had these numbers: 13 29 17 21

24 12. Given below are some responses from learners.

14 30 18 22 25 13 **[Learner 8, 19/02/16]**

15 22 71 22 25 13 **[Learner 5, 23/02/16]**

4.5.4.2 Results Task 8B: Number word before (NWB)

As was the case with NWA, this task had six categories. The task was aimed at assessing learners' abilities in saying NWB in given categories. Learners' responses to NWB are presented in Table 25.

Table 25: Results of learners' abilities in number word before (NWB)

Name of Stage	Number of Learners
Emergent BNWSs to 10	2
Initial BNWSs up to 10	1
Intermediate BNWSs up to10	1
Facile with BNWSs up to 10	2
Facile with BNWSs up to30	2
Facile with BNWSs up to100	4
Total	12

The results in Table 25 above show that 4 of the learners were facile in saying NWB up to 100. On the other hand, 2 learners were still in the emergent category of NWB to 10. The rest of the learners were in between with 4 learners facile with NWB to 10 or above. As compared to NWA task, Table 25 reveals that learners found NWB tasks a bit challenging. One of the categories was “Facile: 1 to 100” and had these numbers: 87 48 61 32 55 99. The following responses are representative of learners' responses to this task.

86 47 60 54 98 30 **[Learner 7, 23/02/16]**

No attempt **[Learner 9, 18/02/16]**

4.6 Results from lesson observations

As stated in Chapter 3, mathematics lessons in number concepts were observed, and each of the 5 teachers was observed once. The lesson observation schedules targeted specific areas which are presented in the subsections that follow.

4.6.1 Types of resources used in the teaching and learning of early number concepts

One aspect that the study explored was types of resources that mathematics teachers used to teach early number concepts. Table 26 below summarises what was observed regarding types of resources teachers used to teach early number concepts.

Table 26: Types of resources used in teaching early number concepts

Types of teaching and learning resources	Number of teachers
Counters	5
Place value box	2
Charts	2
Number cards	1
Number tree	1
Pictures	1

Table 26 above shows that all 5 teachers used physical objects such as sticks, 1 teacher in each case used pictures, number tree and number cards and 2 teachers were also observed using place value box and charts in teaching early number concepts. Pictures, number

cards and number tree were observed in Standard 1 teachers while place value boxes were used by Standard 2 teachers. These findings correspond with what teachers reported during interviews in which all of them indicated that they use real objects to teach early number concepts. Figure 8 below shows some resources used in the teaching and learning early number concepts which include counters among others.



Standard 2B

Figure 8: Types of teaching and learning resources for early number concepts

4.6.2 Where teaching and learning resources were placed, who used them and how they were used during mathematics lessons

The researcher observed where teaching and learning resources were placed in the classroom, including who used them between the teacher and the learners and how they were used. It was noted that all the 5 teachers had teaching and learning resources on their desks such as counters and place value boxes. Standard 1 learners had counters

brought from their homes daily whereas counters for Standard 2 learners were stored in their classrooms. Thus, during mathematics lessons, both teachers and learners had counters which they used to model counting, addition and subtraction tasks together. The extract from one of the lessons given below represents a typical mathematics lesson:

Teacher: ... *Chabwino. Ndiye pano tikufuna ma lidala aja; amene ali lidala apite akatenge mawererengero paja timasunga paja. Ma lidala katengeni mawererengero, katengeni mawererengero a wina aliyense kuti tigwiritse ntchito, tikatha tibwezeretsa....* (Now, we want all group leaders, those that are group leaders to go and collect counters for everyone from the usual storage place. After using them, we will put them back ...).

Teacher: *Aliyense watenga mawererengero, eti?* (Has everyone got counters?)

Class: *Eeee* (Yes)

Teacher: *Chabwino* (Right). (Writes the following example on the chalkboard)

$$\begin{array}{r} \text{T} \quad \text{O} \\ 7 \quad 8 \\ - \quad \quad \\ 5 \quad 1 \\ \hline \end{array}$$

: *Taona kutsogolo kuno?* (Have we all seen what is written here?)

Class: *Eeee* (Yes)

Teacher: *Ndiye poyambirira tione kuti 78 tichotsepo zingati, fifite chiyani apa?* (First, what do we subtract from 78?)

Class: 51

Teacher: *Ndiye tiyamba kuchotserana ma ones achotserana ma chiya? Ma ones achotserana ma chiyani? Ma ones okhaokha, eti? Kenako, ma tens achotserana ma chiyani?* (What do we subtract ones from; ones versus ones, okay? Then, tens subtracting what?)

Class: *Ma tens okhaokha* (Tens subtracting tens only)

Teacher: *Aliyense apezake ali ndi chiyani?* (What should everyone have?)
 Class: *Mawerengero* (Counters)
 Teacher: *Mawerengero. Tikumvana eti? Chabwino. Tiyeni tiyambepo. Apa pali zinthu zingati?* (Counters. Is it clear? Fine. Let's start. How many objects are in this number?)
 Class: 8
 Teacher: *Chabwino. Ndiye tiyeni tiyambe kuwerenga* (Right. Now, let's start counting)
 All: 1, 2, 3 ... 8.
 Teacher: *Tinati zinthu zingati?* (How many objects were we supposed to count?)
 Class: 8

[Teacher E; Standard 2A; 27/01/16]

4.6.3 Teaching methods and strategies used to teach and learn early number concepts

A method, in this study, is used to mean a way of teaching learners, such as group work, question and answer and individual work. A strategy, on the other hand, is used broadly, to mean any procedure used to solve an arithmetic problem that can result in a correct answer, such as counting on fingers, retrieving the answer from memory, mentally calculating the answer to the problem, or using the algorithms taught in classrooms (Carr & Alexeev, 2011). It was observed that all 5 mathematics teachers used demonstration, question and answer, group work, pair work and also individual work. In addition, 4 of the teachers used group work whereas 2 teachers used songs to teach specifically number counting and identification. In all these instances, both teachers and learners used counters to model number counting, addition and subtraction.

When doing individual work, teachers indicated that learners should not discuss but do the work on an individual basis. In spite of this, some learners were seen discussing especially when the teacher faced the other side such that he or she could not see them.

All the 5 teachers taught in the same way as their lessons were divided into three similar phases. In phase 1, the whole class and the teacher did an example together using counters. In Phase 2, learners were given an exercise to do in groups, and, finally, in Phase 3, learners were given work to do individually. Teachers reported that this is how they were trained to teach these early grades under the early grade reading activities (EGRA) intervention.

4.6.4 Learners' experiences and activities in mathematics lessons

The study noted that all 5 mathematics teachers engaged learners in modeling counting, addition and subtraction of numbers as well as numeral identification. In addition, 4 of the teachers provided learners with activities of writing numbers on the chalkboard and in their exercise books. Lastly, it was also observed that 1 teacher provided learners with number picking activities on number cards. Thus, in general, learners' activities and experiences in mathematics lessons included modeling of number concepts.

4.6.5 Learners' strategies in early number concepts

The study found that in all the 5 lessons observed, learners used counters and their fingers to model numerical problems. When given work to do in groups, pairs, or individually, most learners were seen using their counters while others were seen counting their

fingers. Some learners were also seen moving their lips suggesting the use of oral counting. Yet other learners (though rarely) could just wait for the teacher to ask them to give an answer to a given task. These learners were not seen doing anything. When presenting work from their groups to the whole class, group representatives were seen demonstrating using counters to explain how their group got answers to given tasks. These findings relate to what some learners were doing during oral assessment interviews and also what their teachers reported during interviews.

4.6.6 Types of questions teachers asked during mathematics lessons

The study found that teachers mixed both high (why and how) and low (what) order questions. They mostly posed higher order questions when they wanted learners to demonstrate, with counters, how and why they got an answer to a number problem. This was observed when learners were presenting work from their groups, or when they were handing in individual work for marking. Teachers first asked the learner to demonstrate with counters how she or he got an answer to a problem before marking it. Teachers reported that doing so helps them establish whether the learner has understood a particular concept or not, and also to check those who have just copied solutions from fellow learners. Upon asked why learners could not just explain how they got their answers, all the 5 teachers reported that learners could not manage to explain in the absence of counters because they were, something which undermined learners' abilities.

Teachers asked low order questions especially when they needed just an answer to a given number task from learners. For example, one teacher asked learners this question: 4

kuchotsa 1 yankho lake ndi chiyani? (How much is 4 take away 1?). So, all the 5 teachers used low order questions most frequently while 4 also used high order questions as well. All 5 teachers also gave explanations to stress a point or correct learners' thinking to number problems they got wrong.

4.7 Results from analysis of documents and other visual texts

The researcher analysed teachers' schemes and records of work, lesson plans, classroom resources and children's exercise books for information related to the focus of the study. The sections that follow present what the study found in these documents.

4.7.1 Teaching and learning methods indicated in mathematics teachers' schemes and records of work

In schemes and records of work, all 5 mathematics teachers indicated demonstration, group work, individual work and question and answer for teaching early number concepts. Further to this, 2 teachers in each case indicated games, songs and pair work whereas 1 teacher included discussion. All these methods were also observed during mathematics lessons and reported during teachers' interviews.

4.7.2 Teaching methods found in mathematics teachers' lesson plans

Same methods noted in teachers' schemes and records of were also indicated in indicated in lesson plans. Table 27 presents a summary of the teaching methods teachers indicated in their lesson plans.

Table 27: Teaching methods indicated in mathematics teachers' lesson plans

Teaching method	Number of teachers
Demonstration	5
Question and answer	5
Group work	4
Explanation	3
Songs and Games	2
Individual work	2
Observation	1
Discussion	1

As was the case with schemes of work, Table 25 reveals that all 5 teachers indicated demonstration and question and answer with 4 of them indicating group work and 3 others explanation. These methods were also observed from all 5 teachers during their lesson. Such methods allow the teacher to do more talking in the course of leading learners. During interviews, teachers reported that learners in these classes are young and learn by copying from the teacher. Although not all teachers indicated such methods as discussion, observation and individual work, all teachers reported during interviews that they use them, and indeed all were observed using all of these methods at some points during mathematics lessons. Figure 9 shows a sample lesson plan showing teaching methods teachers indicated.

TEACHERS' ACTIVITIES	LEARNERS' ACTIVITIES		
DEVELOPMENTAL STEPS			
Step 1		Step 4	
Let learners to be in groups	being in groups	ask learners to open on page 31 and write down the numbers each illustration represents	opening page 31 and writing the numbers
Give learners place value box	receiving value box and counters	CONCLUSIONS	
Step 2		Marking learners' work and making corrections	making corrections
model the number 30 together with learners	modelling 30 together with the teacher	LESSON EVALUATION	
discuss with them how to form numbers 31 up to 39	discussing with the teacher how to form numbers 31 up to 39	The lesson was successful because learners have achieved done well in the lesson activities. This is so because of adequate resources and time.	
Step 3			
Write the numbers	reading the		

[Teacher E, Standard 2A, 27/01/16]

Figure 9: A sample lesson plan showing teaching and learning methods

4.7.3 Learning experiences and activities for early number concepts

indicated in teachers' schemes of work

The study found that all 5 mathematics teachers who took part in the study indicated the following in their schemes and records of work: modelling numbers in counting, addition and subtraction, tracing or writing numbers, arranging numbers in order of size, completing number sequences, oral counting and reading of numbers, completing

addition and subtraction sentences, adding and subtracting numbers, and writing addition and subtraction sentences. The difference, however, was that Standard 1 dealt with single-digit numbers (0 to 9) while Standard 2 went up to two-digit numbers (0 to 99) as per mathematics syllabi for these grades in Malawi (MIE, 2007). During interviews and lesson observations, these lesson experiences and activities also came out.

4.7.4 Learning experiences and activities for early number concepts indicated in mathematics teachers' lesson plans

It was noted that all 5 mathematics teachers same lesson experiences and activities which were noted during interviews, lesson observations and in schemes of work. Figure 10 is a typical lesson plan showing these learning experiences and activities.

read number 70 to the learners • read number 70 together with learners from the number card • ask learners to read number 70 individually	listening • reading number 70 • reading number 70														
Step 2 • model number 70 using place value box eg <table border="1"> <tr> <td>7</td> <td>0</td> </tr> <tr> <td> </td> <td></td> </tr> <tr> <td> </td> <td></td> </tr> <tr> <td> </td> <td></td> </tr> <tr> <td> </td> <td></td> </tr> <tr> <td> </td> <td></td> </tr> <tr> <td> </td> <td></td> </tr> </table>	7	0													observing
7	0														
model number 70 using place value box sticks together with learners • ask learners to model number 70	modelling number 70 • modelling number 70														

[Teacher D, Standard 2B, 25/01/16]

Figure 10: A mathematics lesson plan

As noted in Figure 11, the lesson experiences and activities include modeling, tracing or writing numbers, arranging numbers in order of size, completing number sequences among others.

4.7.5 Teaching and learning resources found in teachers' schemes and records of work

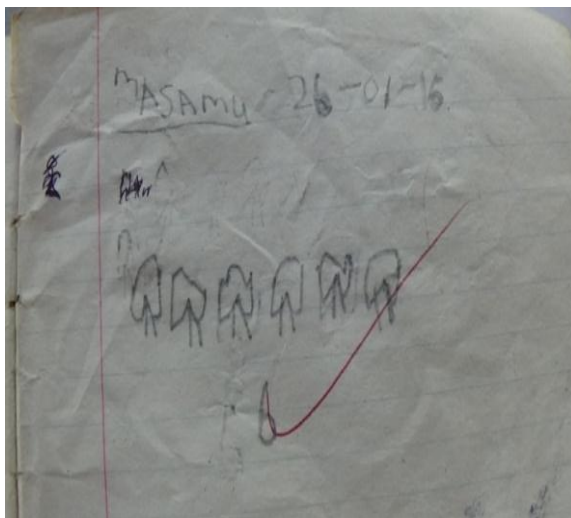
The study found that all 5 mathematics teachers indicated such teaching and learning resources charts, counters like sticks and number cards. In addition, 4 of the teachers indicated songs while 3 others (all Standard 1) indicated number game and number tree. Further to that, 2 indicated had place value box whereas 1 indicated pictures. All these resources were also revealed during interviews and lesson observations.

4.7.6 Teaching and learning resources found in mathematics teachers' lesson plans

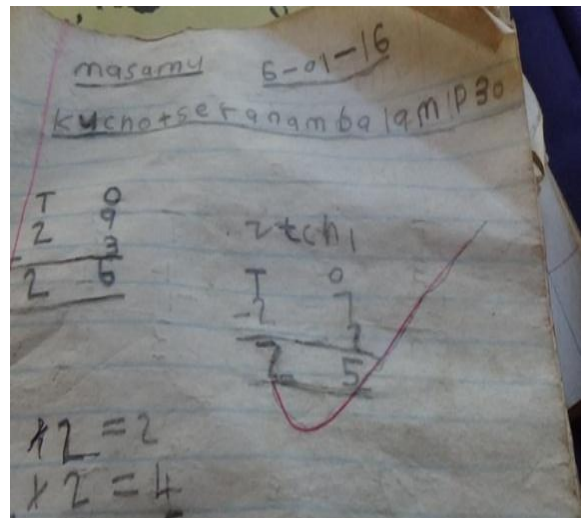
It was noted that all 5 teachers indicated counters or real real objects and number cards. Further to that, 4 of the teachers also indicated songs while 3 teachers indicated number games and number tree. Added to this, 2 (both Standard 2) of the teachers teachers indicated value box whereas 1 teacher indicated pictures. Thus, the study found that teachers indicated same teaching and learning resources revealed during interviews and lesson observations.

4.7.7 Learning experiences and learners' understanding of early number concepts found in learners' exercise books

In the learners' exercise books, the following lesson experiences were noted: modeling counting, addition and subtraction, writing and/or tracing numbers, arranging numbers in order of size, and writing and completing number sequences. Typical of what the learners were doing is shown in Figure 11.



Learner 3; Standard 1B



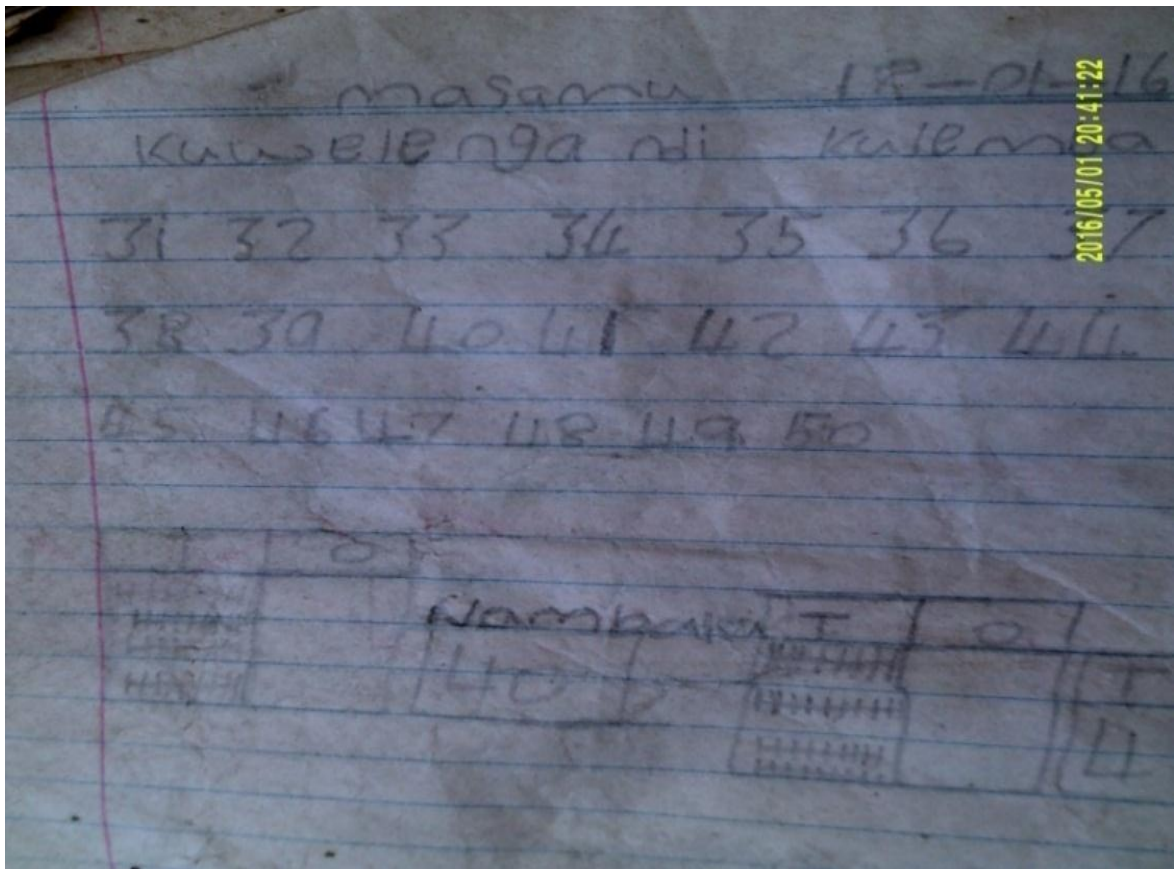
Learner 10; Standard 2A

Figure 11: Learning experiences and learners' understanding of early number concepts

As is shown in Figure 11 above, the lesson experiences were also revealed in teachers' schemes of work and lesson plans as well as during interviews and lesson observations, where learners ended up finding the total number in each of the three early number concepts explored in this study.

4.7.8 Learning resources found in learners' exercise books

In terms of resources, learners' exercise books had resources which were also noted in teachers' schemes of work and lesson plans as well as classroom resources. Figure 12 shows some of the resources noted in this regard.



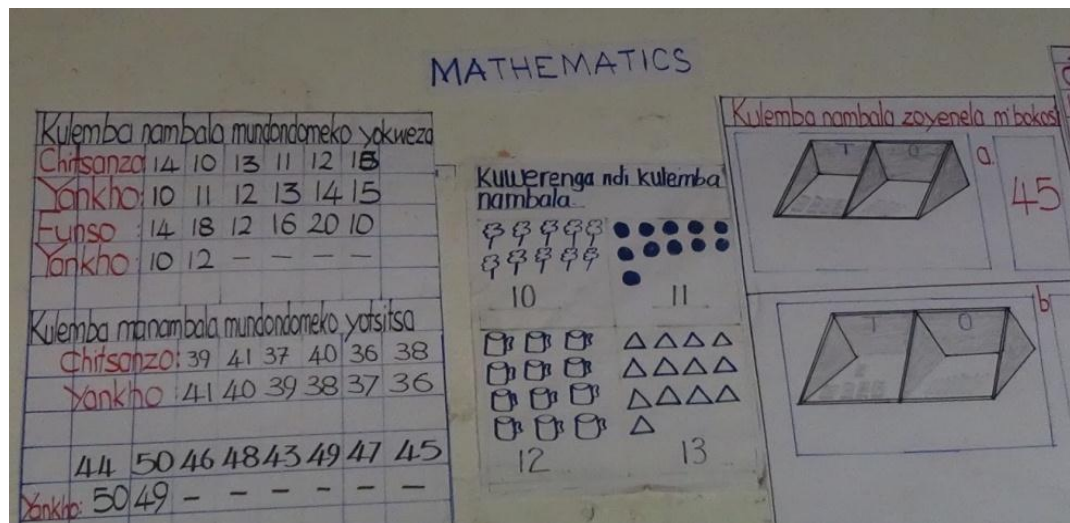
[Learner 7; Standard 2A]

Figure 12: Learning resources found in learners' exercise books

As can be observed from Figure 12, learners used same resources as their teachers such as counters and place value box, slashes and pictures especially Standard 1 exercise books.

4.7.9 Teaching and learning resources found in the classroom

Mathematics resources that were found in the classroom include counters (Figure 8 on page 140), place value boxes, charts, moulded clay numbers and number trees. Figure 14 shows other resources found in mathematics classrooms.



[Standard 2A]

Figure 13: Classroom resources for teaching and learning number concepts

The resources in Figure 13 suggest that learners are involved in number identification, modeling counting, addition and subtraction and tracing or writing numbers just to mention but a few. All teachers involved in the study also reported during interviews that they use such resources for modeling number concepts. In addition, some of the resources like place value box were noted in teachers' schemes of work and lesson plans. Finally, learners also used similar resources during oral assessment interviews and in their exercise books.

4.8 Discussion of findings

This section discusses the findings of the study presented in the first section of this chapter. The study was guided by the following main research question: How do learners in lower classes in Malawian primary schools learn number? To answer this question, four sub research questions were used. The first sub research question was: What strategies do mathematics teachers use to teach early number concepts (counting, addition and subtraction)? The second one was: How do learners understand number concepts (counting, addition and subtraction)? The third sub research question was: What strategies do learners use to count, add and subtract numbers? The fourth and last sub research question was: What levels have learners reached in basic numeracy (forward and backward number word sequences, numeral identification, number word after and number word before)? The findings are discussed in the order in which the research questions appear.

4.8.1 Teachers' strategies in early number concepts (counting, addition and subtraction)

From interviews, the study revealed that all 5 mathematics teachers gave learners learning experiences that engaged them in modeling counting, addition and subtraction using physical objects. Use of real objects in modelling early number concepts was also observed during lesson observations. Added to that, analyses of teachers' schemes of work and lesson plans showed that teachers indicated learning experiences that engaged learners in modeling the early number concepts under study.

In addition, mathematics classrooms had counters and charts containing models of early number concepts. The study, therefore, found that mathematics teachers use counting strategies to teach early number concepts.

Counting strategies use physical objects such as blocks, fingers or marks and oral or verbal counting to model numerical problems (Baroody, 2006; Baroody, Bajwa & Eiland, 2009). In so doing, children are able to link real quantities with the counting numbers, an important step towards children's ability to construct a rich set of relationships among the three worlds of mathematics, namely quantities, counting numbers and formal symbols (Griffin, 2004).

These findings relate to Askew (2013), Fuson (1992b), Fuson et al (1997), Baroody (2006), and Baroody, Bajwa and Eiland (2009) who also found that in learning multi-digit addition and subtraction, children start by using counting strategies to help them deal with such situations. In addition, these findings are consistent with Wright et al (2006)'s (Table 2 on page 65) conceptual framework in which children use such strategies during their initial stages of experience with number concepts. By using counting strategies in teaching and learning early number concepts, abstract number and simple arithmetic ideas are made concrete (Baroody, 1985; Ojose, 2008), making these mathematical concepts accessible to learners. However, counting strategies tend to be slower and less efficient (Siegler & Robinson, 1982; Siegler & Shrager, 1984).

4.8.2 Learners' understanding of early number concepts (counting, addition and subtraction)

From interviews with mathematics teachers, 4 out of the 5 teachers reported that learners understand counting as number of things in a group, addition as number of objects in a collection after mixing them, and subtraction as number of objects that remains in a group after taking away some or all of them from the group. From oral assessment interviews with learners on one-to-one correspondence counting, the study found that 4 out of 12 learners correctly made their maximum counts in the range 101 to 120. Added to this, they all correctly indicated the number of circles they had counted. In addition, 3 out of 12 learners counted correctly in the range 21 to 40. Finally, 1 out of 12 learners also correctly counted in the range 61 to 80. All this means that learners understood what counting was all about although cardinality principle had not yet developed among few of them.

From the learners' oral assessment interviews, 11 out of 12 and all the 12 learners were able to carry out processes of addition and subtraction tasks. The study, therefore, found that learners understood counting as cardinality of objects in a given collection, and addition as an increased cardinality of objects in a collection as a result of putting objects together. On the other hand, learners understood subtraction as a reduced cardinality of objects as a result of removing some or all objects from a given collection. These findings are consistent with Wright (2013) and Wright, Martland and Stafford (2000) who contend that children do counting when they use a number word sequence (FNWS or BNWS) in a context that involves associating number words with some sense of quantity. In addition, the findings relate to Gelman and Gallistel (1978) and Ginsburg (1977) who contend that

children's fundamental understanding of addition and subtraction evolves from their early counting experiences in which they recognise that adding something to a collection makes it larger and taking away something from the collection makes it smaller.

Further to the above, the present study's findings on children's understanding of early number concepts relate to Hartnett and Gelman (1998)'s and Sarnecka and Gelman (2005)'s study findings that children know that the operations of adding and subtracting systematically increase and decrease the value of cardinality, even if they cannot reliably count the number of items involved. As such, children construct an informal conceptual basis for understanding addition as an incrementing process and subtraction as a decrementing process (Baroody & Wilkins, 1999; Gelman, 2006).

4.8.3 Learners' strategies in counting, addition and subtraction of numbers

From interviews, all the 5 teachers reported that learners use real objects like sticks to model early number concepts. From learners' oral assessment interviews, 6 out of 12 did answered addition and subtraction word problems without using physical objects or verbal counting. In addition, 4 learners used count-all while 1 used count-on strategies. The twelfth learner used guessing. On addition number problems, 5 learners used count-all, 3 learners used decomposition, 2 learners used count-on, and 1 learner used mastery strategies. Lastly, the twelfth learner used guessing. On subtraction problems, 10 out of 12 learners used count-all and 1 learner in each case used decomposition and mastery strategies. From analyses of learners' exercise books, all 12 learners used models, slashes or pictures like place value box to workout number problems.

These findings reveal that learners use three types of strategies in learning counting, addition and subtraction of numbers namely; counting, reasoning and mastery. Each one of these types of strategies is discussed in detail in the next sub sections.

4.8.3.1 Counting strategies

Counting strategies rely on the use of physical objects or verbal counting as a way of solving numerical problems (Baroody, 2006; Baroody, Bajwa & Eiland, 2009). Fuson (1992b) argues that children use these physical objects to model directly the addition or subtraction operation given in the situation. Children count all the objects to add, and they take away and count the remaining objects to subtract. In so doing, counting strategies put abstract number and simple arithmetic concrete (Baroody, 1987; Ojose, 2008). As such, children easily access these mathematical concepts, and are able to link real quantities in time and space with the counting numbers, which is an the first step towards children's ability to relate the three worlds of mathematics, namely quantities, counting numbers and formal symbols (Griffin, 2004).

In their studies on children's strategies in multi-digit number addition and subtraction, Askew (2013), Fuson (1992b), Fuson et al (1997), Baroody (2006), and Baroody, Bajwa and Eiland (2009) also found that children use counting strategies first before progressing to more sophisticated ones. The use of counting strategies relate to the conceptual framework which informed this study (Wright et al., 2006) (Table 2 on page 65) in which counting strategies are shown to be used first as children progress to more advanced strategies.

In addition, with respect to cognitive development theories as proposed by Piaget and Ausubel, younger children rely on the use manipulatives to make sense of mathematical concepts.

Despite helping learners make fewer mistakes, counting strategies tend to be slower and less efficient (Siegler & Robinson, 1982; Siegler & Shrager, 1984). This is particularly the case where large numbers are involved. In addition, the study sample comprised younger and older children, who were at different levels according to Piaget's stage theory of cognitive development.

4.8.3.2 Reasoning strategies

Reasoning strategies are built on the counting strategies on Wright et al (2006)'s conceptual framework. In their calculations, learners use known information like known number facts and relationships to logically deduce the answer of an unknown number combination (Baroody, 2006; Orton, 2004; Baroody, Bajwa and Eiland, 2009). Learners do this by first realising that given quantities are composed of ideal chunkable unit items that can be combined and separated in flexible ways (Fuson, 1992b). Then, children separate one or both numbers in a numerical problems commonly into doubles (Fuson, 1992b) like $5 + 6 = 5 + 5 + 1 = 10 + 1 = 11$ (Askew, 2013), or recompose them into ten-structured triplets (Fuson, Stigler & Bartsch, 1986; Fuson & Kwon, 1992a) such as $7 + 6 = 7 + 3$ (to make ten) $+ 3 =$ ten three (13).

Reasoning strategies relate to children's decomposition strategies (Askew, 2013), structuring numbers (Treffers 2001; Ellemor-Collins & Wright, 2009), use of part-whole

relations (Zhang, 1988; Zhang & Liu, 1991), composition or decomposition of numbers (Lin, 1994). On the conceptual framework by Wright et al (2006) that informed this study, these reasoning strategies relate to children's use of facile strategies in which children use a range of what are known as non-count-by-ones strategies and involve procedures other than counting-by-ones but may also involve some counting-by-ones. They use such strategies as compensation, using a known result, adding to ten, commutativity, subtraction as the inverse of addition, and awareness of the 'ten' in a teen number among others (Wright et al., 2006).

Fuson et al (1997) contend that learners find it easier to add than to subtract because addition involves a positive action whereas subtraction involves a negative action, and is problematic for most learners. In addition, they assert that subtraction becomes easier only when learners have fully mastered addition.

4.8.3.3 Mastery strategies

With mastery strategies, learners produce answers to number problems in an efficient-fast and accurate- manner (Baroody, 2006; Baroody, Bajwa & Eiland, 2009). They do so without the aid of counters or their fingers. Askew (2013) refers to such strategies as retrieval strategies. He contends that children can recall an answer from their memory within 3 seconds and further adds that children gradually progress from counting strategies, to decomposition strategies, and to retrieval strategies. This progression of children's strategies for number addition and subtraction is also reflected from Wright et al (2006)'s conceptual framework. Over the course of development, children replace slow counting procedures and thinking strategies with rapid fact retrieval (Ashcraf, 1982).

Given an infinitely large arithmetic system, the use of such automatic reconstructive processes would make sense-would be cognitively economical (Baroody, 1985). Accurate and automatic production of the basic number combinations' is a major objective of elementary mathematics education. Basic number combinations will refer to the 100 addition combinations with single-digit addends ($0 + 0$ to $9 + 9$) and the 21 combinations in the series $10 + 0$ to $10 + 10$ (including their commuted pairs). It will also refer to the corresponding subtraction, multiplication, and division combinations. Further, it relates to Wright, Martland and Stafford (2000)'s "automised" or "habituated" strategies (P. 30) in which children produce answers to numerical problems through combining and partitioning of small numbers (structuring numbers).

4.8.4 Learners' levels in basic numeracy

The study explored five aspects of basic numeracy namely; forward and backward number word sequences (FNWSs and BNWSs), numeral identification (NI), number word after (NWA) and number word before (NWB). Numbers in these tasks were selected while also taking into account special numbers like doubles, decades and those with 1s and 0s in the 'ones' column. Learners have problems with these numbers (Wright, Martland & Stafford, 2006). Each of these levels is discussed in detail in the sub sections that follow.

4.8.4.1 Learners' levels in forward number word sequences (FNWSs)

To determine children's levels in FNWSs, the study used the model for FNWSs (Table 4 on pages 68). Thus, the study found that 6 out of 12 learners were on Level 5 meaning that they were facile in saying FNWSs involving one-and two-digit numbers up to 100. In

addition, 3 learners were on Level 4, containing one- and two-digit numbers up to 30. Finally, 1 learner in each case was Levels 3, 2, and 1. This implies that at the time of this study, every learner knew to say FNWSs beyond Level 0 -the Emergent FNWSs to 10.

4.8.4.2 Learners' levels in backward number word sequences (BNWSs)

To find learners' levels in BNWSs, the study used a model for BNWSs (Table 5 on pages 70). In this case, 5 learners were on Level 4 (Facile BNWSs to 30), 2 learners in each case were on Levels 3, 2 and 0 whereas 1 learner was on Level 1. As compared to the learners' abilities in saying FNWSs, these results show that learners found BNWSs a bit challenging since no learner had reached Level 5.

4.8.4.3 Learners' levels in numeral identification (NI)

Children's levels in NI were arrived at using a model for NI (Table 3 on page 66). It was, therefore, found that 5 out of 12 learners were on Level 3, 1 learner on Level 2, and 4 learners on Level 1. Finally, 2 learners were still on Level 0.

4.8.4.4 Learners' levels in number word after (NWA)

Children's levels in NWA were determined using a model for FNWSs (Table 4 on pages 68) because NWA and FNWSs tasks are related. Thus, the study found that 7 out of 12 learners were on Level 5, 2 learners on Level 1, 1 learner in each case on Levels 3 and 2, and 1 learner still on Level 0. These results, therefore, suggest that generally, learners found NWA task more challenging as compared to FNWSs task.

4.8.4.5 Learners' levels in number word before (NWB)

Children's levels in NWB were determined using a model for BNWSs (Table 5 on pages 70) as the two tasks are related. So, the findings were that 4 learners were on Level 5, 2 learners in each case on Levels 4 and 3, 1 learner in each case on Levels 2 and 1, and 2 learners still on Level 0. These findings generally reveal that learners found NWB task a bit challenging as compared to NWA task. In NWA task, there is an implicit addition involved just like FNWSs while NWB involves a negative action on the numbers just like in BNWSs. Subtraction is difficult for learners (Fuson et al., 1997).

4.9 Challenges faced

The study identified challenges which teachers and learners face in the course of teaching and learning early number concepts. These challenges are discussed in the next sub sections.

4.9.1 Teachers' challenges

The study found that all 5 teachers in the study reported that learners lack background knowledge in number operations. In addition, 3 of the teachers reported that learners are frequently absent from school while 2 teachers in each case complained of large classes and playfulness on the part of some learners. Finally, 1 teacher in each case reported problems of too much content in the mathematics curriculum, lack of interest of some learners and few learning hours per day.

On learners' background knowledge, studies have revealed that children start formal schooling with lots of informal mathematical knowledge constructed throughout their

first five years of life at home (Kilpatrick, Swafford, & Findell, 2001; Askew, & Wiliam, 1995; Gervason, 2007; Perry & Dockett 2002; Clements & Sarama, 2009) which could serve as input knowledge to their learning of number concepts in these lower classes. This probably suggests that much teaching and learning of early number concepts does not incorporate what these learners already know, and creates a bit of mismatches in the process.

Findings on few school learning hours per day concur with Brombacher (2011) and USAID (2014) who also observed that the primary school day in Malawi is much shorter than most countries in the SADC region (starting at 7:30am and ending at 10:40am) giving time on task close to three hours only. This is particularly the case in lower primary school and this gives limited time for teachers to give intensive instruction. As such, teaching and learning of early number concepts is negatively affected as teachers need to rush through in order to cover as much ground as possible. In the process, teachers end up giving children little time on task.

In Malawi, the problem of large classes stems out of high enrolment in primary school education as a result of free primary education (FPE) which the Government of Malawi introduced in 1994 (World Bank, 2010; Kunje, Selemani-Meke, & Ogawa; 2009). The introduction of FPE in Malawi also resulted in shortage of classrooms, teaching and learning resources and qualified teachers. However, by 2004, some improvements such as a decrease in the number of untrained teachers and provision of text books were made (Kunje, Selemani-Meke & Ogawa, 2009).

In their study on effects of large class size on effective teaching and learning at the Winneba Campus of the University of Education, Winneba (UEW), Ghana, Yelkperi et al. (2012) found that students reported that large class size did not afford lecturers an opportunity to pay attention to weaker students and do remedial teachings. Similarly, in London, a study by Blatchford, Bassett and Brown (2011) on effects of class size on classroom interactions and pupil behaviour found that at primary and secondary levels, smaller classes led to pupils receiving more individual attention from teachers, and having more active interactions with them. In addition, the study found that classroom engagement decreased in larger classes. All this indicates that large class sizes are a problem to teachers as they reduce interactions between teachers and learners and teachers' attention to individual learners. If learners are not adequately assisted, they may not benefit much from classroom instruction thereby hampering their learning of early number concepts.

4.9.2 Learners' challenges

On learners' challenges, 4 out of 5 teachers reported that learners are frequently absent from school whereas 2 teachers in each case reported of large classes, inadequate teaching and learning resources and learners' lack of background knowledge in number operations. Finally, 1 teacher in each case reported that some teachers use poor teaching methods, some learners are lazy and/or playful, and some are slow learners while other learners are underage.

Teachers asserted that most learners in Standards 1 and 2 are under-age. This contradicts a study by World Bank (2016) which found that a large number of learners who enter

primary school in Malawi are over-age. According to World Bank, however, the proportion of over-age pupils in Standard 1 has declined slightly over the course of the past decade, from 55.8 to 49.4 percent. This means that the problem of under-age children in Malawian primary schools in general and Standards 1 and 2 in particular is not as serious as teachers in this study claimed. What needs to be done, therefore, is to check learners' prerequisite knowledge in number concepts and find better ways of assisting them.

4.9.3 Solutions to the challenges faced

In view of the challenges faced, the study identified a number of solutions which teachers employed. In this case, 2 out of 5 teachers in each case invite parents and guardians for a discussion, teach learners in groups other than individuals, use more able learners to lead such groups and also teach fellow group members, give learners homework frequently, improvise teaching and learning resources, and provide assistance to individual learners. Finally, 1 out of 5 teachers in each case employs effective classroom management strategies, uses a variety of teaching, learning, and assessment methods, provides learners with remedial (free, extra) lessons, and gives learners extra, easy work. Provision of remedial lessons and individual assistance to learners is good because they address specific problems that learners are facing. However, in the face of large classes as discussed in the previous sub section, it is difficult to determine whether teachers really provide such assistance to learners.

4.10 Chapter summary

The chapter has presented and discussed the findings of the study which have revealed different types of strategies in children's learning of number, namely; counting, reasoning and mastery strategies. In addition, it has presented challenges faced by both teachers and learners related to children's learning of number. Finally, the chapter has presented ways teachers use to address the challenges faced. The next and final chapter presents a conclusion of the findings presented and discussed in this chapter.

CHAPTER 5

CONCLUSION, IMPLICATIONS AND RECOMMENDATIONS

5.0 Chapter overview

This chapter presents conclusion on the key findings of the study discussed in chapter four. The conclusion and implications are followed by recommendations of the study. The last section of the chapter outlines suggested area for further research.

5.1 Conclusion

The study explored children's learning of number in lower classes in a Malawian primary school. The main research question which guided the study was: How do learners in lower classes in a Malawian primary school learn number? Four critical research questions were explored in relation to the main research question. Summaries of the key findings of the study are given according to the critical research questions.

1. What strategies do mathematics teachers use to teach early number concepts (counting, addition and subtraction)?

Using a conceptual framework called Learning Framework in Number (LFIN) (Wright, Martland & Stafford, 2000; 2006), the study found that mathematics teachers use one type of strategies called 'counting strategies' to teach early number concepts. The

researcher has argued that although the use of counting strategies alone simplifies abstract number concepts and simple arithmetic (Baroody, 1987; Ojose, 2008) and makes mathematical concepts accessible to learners, and also helps learners make fewer mistakes, they tend to be slower and less efficient (Siegler & Robinson, 1982; Siegler & Shrager, 1984). As such, they may not be effective for numerical problems which have large numbers.

The study showed that children's strategies in early number concepts are dependent on their ages as is reflected in cognitive development theories as proposed by Piaget, Vygotsky and Ausubel. As such, teachers need to be aware of this so that they could identify and support as many types of children's strategies as possible. In this way, teachers can challenge learners to work within their zones of proximal development to achieve meaningful learning.

2. How do learners understand number concepts (counting, addition and subtraction)?

The study found that children recognise that the operations of adding and subtracting systematically increase and decrease the value of cardinality (Gelman, 2006; Hartnett & Gelman, 1998), even if they cannot reliably count the number of items involved (Sarnecka & Gelman, 2005). Therefore, the study found that children construct an informal conceptual basis for understanding addition as an incrementing process and subtraction as a decrementing process (Baroody & Wilkins, 1999) the knowledge of which they use to comprehend and solve simple arithmetic tasks or word problems (Gelman & Gallistel, 1978).

3. What strategies do learners use to count, add and subtract numbers?

The study identified three types of strategies children use to deal with early number concepts, namely counting strategies, reasoning and mastery strategies. Counting strategies are those that rely on the use of physical objects or verbal counting to model these concepts (Baroody, 2006; Baroody, Bajwa & Eiland, 2009; Fuson, 1992b), and make abstract number concepts concrete for children to easily understand (Baroody, 1987; Ojose, 2008). As such, learners can make links among the three worlds of mathematics, namely quantities in time and space, counting numbers and formal symbols (Griffin, 2004). However, counting strategies tend to be slower and less efficient (Siegler & Robinson, 1982; Siegler & Shrager, 1984) especially where large numbers are concerned. Therefore, they should be used strengthened with non counting-strategies to make teaching and learning more effective.

Reasoning strategies are built on the counting strategies, when, in the course of their development, children realise that quantities are composed of ideal chunkable unit items that can be combined and separated in flexible ways (Fuson, 1992b). As such, learners mostly use a variety of non-count-by-ones strategies such as part-part-whole relations (Zhang, 1988; Zhang & Liu, 1991), structuring numbers (Treffers 2001; Ellemor-Collins & Wright, 2009), decomposition (Askew, 2013; Lin, 1994), compensation and commutativity (Wright et al., 2006; Orton, 2004; Baroody, Bajwa and Eiland, 2009; Askew, 2013; Fuson et al., 1997; Fuson, Stigler & Bartsch, 1986; Fuson & Kwon, 1992a).

With mastery or automatic strategies, children retrieve solutions to numerical problems from memory efficiently (Askew, 2013; Baroody, 2006; Baroody, 2006; Baroody, Bajwa & Eiland, 2009). Children develop these strategies over their course of development and replace slow counting procedures and thinking strategies with rapid fact retrieval (Ashcraft, 1982). As a result, mastery strategies are cognitively economical and allow a child's brain to concentrate on more complex issues (Baroody, 1985; Baroody, Bajwa & Eiland, 2009).

It is, therefore, observed that as children get mature, they gradually move from counting strategies, to reasoning strategies, and finally to mastery strategies (Ashcraft, 1982; Askew, 2013; Baroody, 2006; Baroody, Bajwa & Eiland, 2009). This order children's acquisition of early number concepts is also reflected in Wright et al (2006)'s conceptual framework which informed this study whereby they gradually move from counting-by-ones through non-counting-by-ones to facile strategies. In addition, this trend also conforms to cognitive development perspectives as proposed by such cognitive development psychologists as Piaget, Vygotsky and Ausubel, where children's thinking gradually and qualitatively moves from manipulating physical objects to abstract thinking while being aided with language as a tool for not only communication but also thinking (Ojose, 2008; Woolfolk, 2007; Ormrod, 2014; Vygotsky, 1978; 1981; Ausubel, Novak & Hanesian, 1978).

4. What levels have learners reached in basic numeracy (numeral identification, forward and backward number word sequences, number word after and number word before)?

On forward number word sequences, 6 learners were on Level 5, 3 learners on Level 4, and 1 learner was each on Levels 3, 2, and 1. On backward number word sequences, 5 learners were on Level 4, 2 learners in each case on Levels 3, 2 and 0, and 1 learner on Level 1. On number word after, 7 learners were on Level 5, 2 learners on Level 1, and 1 learner in each case was on Levels 3, 2 and 0. On number word before, 4 learners were on Level 5, 2 learners in each case on Levels 4 and 3, 1 learner in each case on Levels 2 and 1, and 2 learners were still on Level 0.

Compared to FNWSs task, learners had problems with BNWSs. Similarly, learners found NWB tasks a bit challenging compared to NWA tasks. This is consistent with Wright et al (2000)'s conceptual framework in that FNWSs involve a positive action on the numbers whereas BNWSs are subtractive and involve a negative action which is more difficult for learners.

On numeral identification, 5 learners were on Level 3, 1 learner on Level 2, 4 learners on Level 1, and 2 learners were still on Level 0. Generally, learners found NI tasks easy. This contradicts Wright et al. (2006)'s conceptual framework which indicates that learners have problems identifying two-digit numbers due to inclusion of special numbers like doubles, decades and those ending in 1s and 0s in 'ones' columns.

5.2 Implications

1. Children in lower classes in a Malawian primary school acquire number concepts using a variety of strategies, and, with time, progress from counting, to reasoning, and to mastery strategies (Askew, 2013). This trend is in line with Wright et al (2006)'s conceptual framework.
2. Mathematics teachers in lower primary school at this school undermine learners' abilities by treating them as too young to invent their own strategies in acquiring early number concepts.
3. Mathematics curriculum in lower primary school does not provide adequate learning experiences for older learners. In so doing, it undermines learners' abilities in acquiring early number concepts beyond the use of counting strategies.

5.3 Recommendations

In view of the findings of this study and the implications that have been put forward, it is imperative that a number of issues be looked into.

1. Mathematics teachers should identify and build on what learners already know. This would help them teach using a variety of strategies which learners are already familiar with. In so doing, it would be easy to teach learners within their zone of proximal development to promote learning.

2. Mathematics curricula in lower primary school should provide numerical experiences that require learners to use strategies beyond counting strategies. Such experiences would help accommodate older learners in these classes.

5.4 Area for further research

The findings suggest that teachers need to be well prepared for teaching number. It may, therefore, be worthwhile for other researchers to carry out studies in TTCs to investigate how teacher educators prepare student teachers to teach number concept.

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APPENDICES

Appendix A: Letter to the District Education Manager for Balaka district

Chembera CDSS
Post Office Box 4
Balaka
5th January 2016

The District Education Manager
Balaka District Education Office
Post Office Box 354
Balaka

Dear Sir

SEEKING PERMISSION TO CARRY OUT A RESEARCH STUDY IN PRIMARY SCHOOLS IN BALAKA DISTRICT

I would like to kindly seek permission from your noble office to conduct a research study in primary schools in Balaka district.

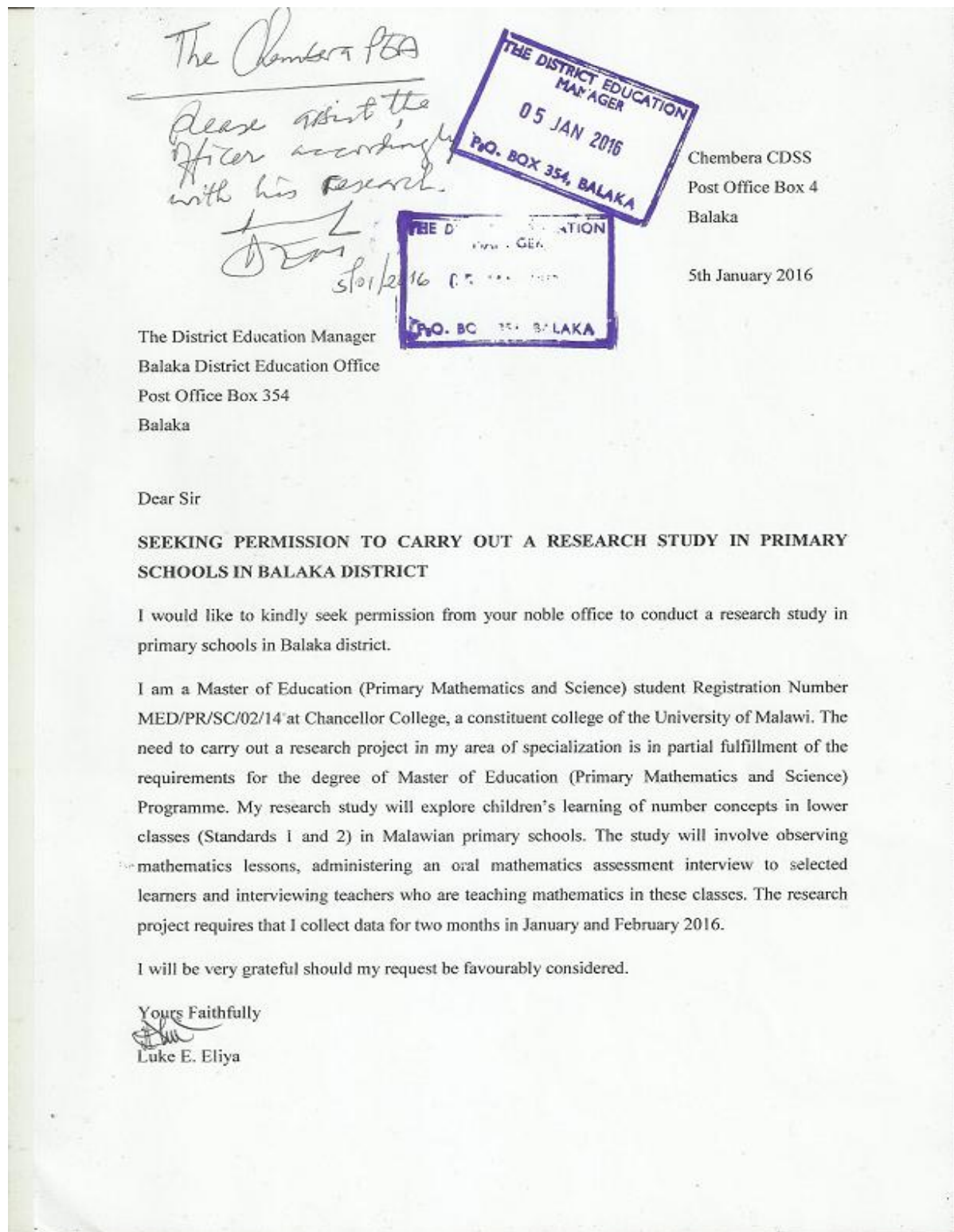
I am a Master of Education (Primary Mathematics and Science) student Registration Number MED/PR/SC/02/14 at Chancellor College, a constituent college of the University of Malawi. The need to carry out a research project in my area of specialization is in partial fulfilment of the requirements for the degree of Master of Education (Primary Mathematics and Science) Programme. My research study will explore children's learning of number concepts in lower classes (Standards 1 and 2) in Malawian primary schools. The study will involve observing mathematics lessons, administering an oral mathematics assessment interview to selected learners and interviewing teachers who are teaching mathematics in these classes. The research project requires that I collect data for two months in January and February 2016.

I will be very grateful should my request be favourably considered.

Yours Faithfully

Luke E. Eliya

Appendix B: Letter from the District Education Manager for Balaka district



Appendix C: Letter of consent for mathematics teachers

Dear Standard 1 / 2 mathematics teacher

I would like to seek consent for your participation in a research project that is part of my Master of Education (Primary Mathematics and Science) Course at Chancellor College, a constituent college of the University of Malawi.

The aim of the study is to explore children's learning of number concepts in infant classes (Standards 1 and 2) in Malawian primary schools. In order to achieve this, learners in these classes will be observed during mathematics lessons and thereafter selected others will be given an oral mathematics assessment. In addition, mathematics teachers handling these classes will be interviewed to give their insights regarding the topic of the study. You are therefore requested to participate in the research project which will be carried out during the months of January and February 2016.

The interview will be reflective in nature and will be based on individual views. With your permission, I would like to video record mathematics classroom observations and also tape record our interview conversations so that I do not forget important information you will provide. The interview will take no more than 50 minutes. I will not use your names when making my report on this research. It is envisaged that your participation in this research project will make you more aware about the nature of children's learning of number concepts in these classes.

If you are agree to participate in the research project, please complete the form overleaf below.

Thank you for your cooperation.

Yours Faithfully

Luke E. Eliya

Consent form for mathematics teachers

I, _____, hereby agree to be a participant of the research project which is to be conducted by Luke E. Eliya. I understand that the purpose of the research is to explore children's learning of number concepts in lower classes (Standards 1 and 2) in Malawian primary schools.

I, therefore, acknowledge that the aims, methods, anticipated benefits and consequences of the research project have been well explained to me. I voluntarily and freely give my consent to my participation in such a study. I understand that results will be used for research purposes. I am free to withdraw my consent at any time during the study if need arises.

Signature: _____ **Date:** ____/____/____

Appendix D: Letter to parent/guardian of selected learners for the study

Chembera CDSS
Post Office Box 4
Balaka
6th January, 2016.

Dear parents/guardians of _____

I write to inform you and seek your permission to allow your child to participate in my research project.

I am studying for a Master of Education (Primary Science and Mathematics) Degree Programme at Chancellor College, a constituent college of the University of Malawi. On top of doing course work, I am required to conduct a research study in my area of specialization. The study will explore children's learning of number concepts in lower classes (Standards 1 and 2) in Malawian primary schools.

Children in these classes will be observed during mathematics lessons. Later on, selected children including your child will be given an oral mathematics interview assessment regarding the issue under study. If you allow your child to take part in the research project, please indicate your consent by completing the form given overleaf.

I appreciate your permission to let your child to be involved in this study.

Yours Faithfully

Luke E. Eliya

Parental Consent Form

I have read the letter about the research project on exploring children's learning of number concepts in lower classes (Standards 1 and 2) in Malawian primary schools and I agree for my child to participate in the study.

Name of the child: _____

Name of Parent /Guardian: _____

Signature of Parent _____

Guardian: _____ **Date:** ____/____/____

Appendix E: Interview guide for mathematics teachers

Teacher's Name ----- Date: -----

Sex: ----- Standard: -----

School's Name: ----- Age (Range): -----

Time: -----

1. How long have you been teaching?
2. How long have you been teaching mathematics?
3. How long have you been teaching mathematics in infant classes (Standards 1 and/or 2)?
4. What teaching methods have you been using to teach learners these number concepts?
 - (i) Counting;
 - (ii) Addition;
 - (iii) Subtraction.

Probe 1: What techniques (activities, learning experiences) are learners involved in?

Probe 2: Which of those techniques seem to help learners grasp those number concepts and why?

5. What strategies do children use to help them do the following actions and/or operations?
 - (a) Counting;
 - (b) Addition;
 - (c) Subtraction.

Probe: What mistakes and misconceptions do learners make and have about counting, addition and subtraction?

- (i) Learners' mistakes about:
 - (a) Counting;
 - (b) Addition;
 - (c) Subtraction.
 - (ii) Learners' misconceptions about:
 - (a) Counting;
 - (b) Addition;
 - (c) Subtraction.
- 6. Are there any children's own strategies which they use to assist them in carrying out each of the following number operations? If so, explain.
 - (a) Counting;
 - (b) Addition;
 - (c) Subtraction.
- 7. How do learners understand?
 - (a) Counting;
 - (b) Addition;
 - (c) Subtraction.
- 8. From your experience as a mathematics teacher, what challenges do you face when teaching children these early number concepts (counting, addition and subtraction)?
- 9. What challenges prevent learners from understanding these number concepts (counting, addition and subtraction)?
- 10. How do you deal with these challenges?

-----We have come to the end of our interview. Thank you for your participation. Your responses were very informative. In case there is something more I may need to know from you regarding this interview; would you allow me to consult you again? -----

Questions 1 to 3 adapted from Susuwele-Banda (2005, p. 153)

Appendix F: Oral assessment interview guide for learners (Chichewa version)

Name of learner: _____	Age: _____	Sex: _____
Standard: _____	Date of Interview: __/__/__	
Name of Class Teacher: _____	Time: _____	

Chilolezo

Dzina langa ndi _____. Ndine mphunzitsi.

- Ndikufufuza m'mene ana amaphunzirira masamu. Iweyo ndiwe m'modzi mwa ana a mwayi omwe asankhidwa. Ndikufuna kuti undithandize. Koma ngati suli womasuka kuthandiza, ndiwe wololedwa kutero.
- Tisewera masewero okhudzana ndi manambala.
- Ndidzigwiritsa ntchito wotchi iyi kuti tione nthawi imene ungatenge poyankha mafunso.
- Awa simayeso a pasukulu pano.
- Ndikufunsanso mafunso okhudzana ndi banja lakwanu monga chiyankhulo chomwe mumayankhula ndi zina zambiri.
- Sindilemba dzina lako, choncho palibe yemwe adziwe zamayankho ako.
- Tiyanbe?

Kodi mwana anavomereza kufunsidwa mafunso? Eya ☐ Ayi ☐

Ngati mwanayo akana kufunsidwa mafunso musamufunse. M'malo mwake, muthokozeni kaamba ka nthawi yomwe anayipereka ndipo itanani mwan wina.

Ngat mwana avomereza kufunsidwa mafunso pitilizani ndi magawo awa:

Mafunso okhudzana ndi kunyumba kwanu

1. Udaphunzirapo ku sukulu ya m'kaka?: _____
2. Kunyumba kwanu mumayankhula chiyankhulide chanji? _____

3. Kunyumba kwanu alipo munthu amene amakuthandiza kulemba ntchito yomwe mwapatsidwa kusukulu? _____
 4. Uli ndi mabuku kapena ma magazine omwe umawerenga kunyumba? _____
 5. Umawonera kanema, kuti? _____
 6. Umamvera wailesi, kuti? _____
-

1. **Task 1: Oral counting**

- Ndikufuna undiwerengere. Ndikuuza nthawi yoyambira ndiyomalizira kuwerengera. Ndiwerengere kuyambira 1 mpakana pamene ungalekezerepo, yamba: 1...
- Umvetsera m'mene ndikufunira kuti uwerengere. 1, 2, 3..., 10. Monga m'mene ndawerengera, werenga kuyambira 1 mpakana pomwe ungalekezerepo, yamba: 1...

Nthawi yomwe mwana watenga powerenga: _____

Nambala yomalizira yomwe mwana wawerengay: _____

2. **Task 2: Counting: One-to-one correspondence counting**

- Ukuwaonawa ndi maseko. Ndikufuna kuti uziloza ndi kuwerenga masekowa. Uyambire seko iyi: 1...
 - Mwana wawerenga maseko angati? _____
 - Mwanayo wanena kuti wawerenga maseko angati _____
 - Nthawi yomwe mwana watenga powerenga maseko _____
-

3.

Task 3: Number identification item - Exercise One

- Ukuwaonawa ndi manambala. Ndikufuna kuti uziloza ndikuitchula nambalayo. Yambira nambala iyi.

				Manambala amene mwana watchula molondola pa mnzere
6	1	19	4	
10	16	3	9	
15	12	7	13	

Kuchuluka kwa manambala onse amene mwana watchula molondola: ____/12

Nthawi yomwe mwana wagwiritsa ntchito: _____

Ngati mwana walephera kutchula manambala anayi pa mnzere umodzi, lumphani Gawo 3 ndikupita ku Gawo.

Task 3: Number identification task – Exercise two

- Ukuwaonawa ndi manambala. Ndikufuna kuti uziloza ndikuitchula nambalayo. Yambira nambala iyi.

				Manambala amene mwana watchula molondola pa mnzere
6	8	18	27	
5	31	47	58	
73	85	99	36	
50	59	73	91	
38	61	76	15	

Kuchuluka kwa manambala onse amene mwana watchula molondola: ____/20

Nthawi yomwe mwana wagwiritsa ntchito: _____

4. **Task 4: Word problems**

- Ndili ndi mafunso oti undipezere mayankho. Ukhoza kugwiritsa ntchito mawererengero awa kapena pepala ndi pensulo izi ngati ukufuna. Ngati sunamvetse funso undiuze kuti ndibwerereze.

Funso loyeserera: John anali ndi switi muwiri (2). Sarah anali ndi switi mtatu (3). Kodi John ndi Sara anali ndi switi mungati yense pamodzi?

Question 1

Jane anali ndi bisiketi 4. Bambo wake ananamupatsa bisiketi wina muwiri (2). Kodi Jane anali ndi bisiketi mungati yense pamodzi?

Question 2

Chisomo anali ndi mango 10. Iye anatenga mango 5 kupatsa Katie. Kodi Chisomo anatsala ndi mango angati?

Question 3

Ana alipo gulu ndipo akusewera. Pagulupo, ana awiri (2) ndi atsikana ndipo 6 otsalawo ndi anyamata. Kodi ndi ana angati omwe akusewera onse pamodzi?

Question 4

Ana okwanira 9 akuyenda kupita kusukulu. 6 mwa anawo ndi anyamata ndipo otsalawo ndi atsikana. Kodi ndi ana agate onse pamodzi amene akupita kusukulu?

5.

Task 5: Addition/subtraction problems
--

- Tsopano tiona masamu owonkhetsa/ophatikiza/a phatikiza ndi ochotsa/ a chotsa.
- Ukhoza kugwiritsa nthito peala ndi pensulo izi kepenso mawerengero awa ngati ukufuna.

a) Masamu owonkhetsa/kuphatikiza/a phatikiza

Instruction: Child can use paper, pencil and/or counters

1. $2 + 3$ yankho lake ndi chiyani?
2. $3 + 4$ yankho lake n'chiyani?
3. $7 + 3$ yankho lake n'chiyani?
4. $9 + 5$ yankho lake n'chiyani?
5. $8 + 7$ yankho lake n'chiyani?
6. $13 + 12$ yankho lake n'chiyani?

b) Masamu ochotsa/ a chotsa/ochotsera

1. $5 - 2$ yankho lake n'chiyani?
2. $7 - 4$ yankho lake n'chiyani?
3. $9 - 6$ yankho lake n'chiani?
4. $15 - 4$ yankho lake n'chiyani?
5. $23 - 7$ yankho lake n'chiyani?
6. $25 - 13$ yankho lake n'chiyani?

Adapted from Malawi Early Grade Mathematics (EGMA Malawi) National Baseline Report (2010) and United States Agency for International Development (USAID) Early Grade Mathematics EDDATA II (2008).

6.

Task 6: Numeral identification

- Ukuwaonawa ndi manambala. Ndikufuna kuti uziloza ndikuitchula nambalayo. Yambira nambala iyi.

Level 1 (Numerals to 10):

9 1 4 0 7 5

Level 2 (Numerals to 20)

13 10 19 12 15 20

Level 3 (Numerals to 100):

23 77 56 98 100 41

7.

Task 7: Number word sequences

(a) Kuwerenga manambala chokweza

Ndondomeko yake: Ndikufuna uwerenge chokweza kuyambira nambal iyi ***, uzingowerenga kufikira ndikuuze kuti ima.

Level 0: (Emergent):

1 (to 10)

Level 1: (Initial: 1 to 10):

1 (to 10)

Level 2: (Intermediate: 1 to 10):

1 (to 10)

Level 3: (Facile: 1 to 10):

1 (to 10)

Level 4: (Facile: 1 to 30):

14 (to 26)

Level 5: (Facile: 1 to 100):

49 (to 67)

81(to 93)

(b) Kuwerenga manambala chotsitsa

Ndondomeko yake: Ndikufuna uwerenge manambala chotsitsa kuyambira nambal iyi *** uzingowerenga kufikira ndikuuze kuti ima.

Level 0: (Emergent):

10 (to 1)

Level 1: (Initial: 1 to 10):

5 (to 1)

9 (to 4)

Level 2: (Intermediate: 1 to 10):

5 (to 1)

9 (to 4)

Level 3: (Facile: 1 to 10):

5 (to 1)

9 (to 4)

Level 4: (Facile: 1 to 30):

17 (to 13)

25 (to 19)

Level 5: (Facile: 1 to 100):

83 (to 74)

61 (to 52)

8.

Task 8: Number word after and number word before

(a) Kutchula manambala chokweza

Tsopano ndizitchula nambala ndipo iwe uzindiuza nambamla yomwe imakbwera kutsogolo kwa nambala ndatichulayo.

Level 0: (Emergent):

7 5 0 4 9 1

Level 1: (Initial: 1 to 10):

7 5 0 4 9 1

Level 2: (Intermediate: 1 to 10):

7 5 0 4 9 1

Level 3: (Facile: 1 to 10):

7	5	0	4	9	1
---	---	---	---	---	---

Level 4: (Facile: 1 to 30):

13	29	17	21	24	12
----	----	----	----	----	----

Level 5: (Facile: 1 to 100):

59	77	82	44	99	31
----	----	----	----	----	----

(b) Kutchula nambala chotsitsa

Tsopano ndizitchula nambala ndipo iwe uzindiuza nambamla yomwe imakbwera kumbuyo kwa nambala ndatichulayo.

Level 0: (Emergent):

2	7	5	4	9
---	---	---	---	---

Level 1: (Initial: 1 to 10):

2	7	5	4	9
---	---	---	---	---

Level 2: (Intermediate: 1 to 10):

2	7	5	4	9
---	---	---	---	---

Level 3: (Facile: 1 and 10):

2	7	5	4	9
---	---	---	---	---

Level 4: (Facile: 1 to 30):

26	18	11	23	14	20
----	----	----	----	----	----

Level 5: (Facile: 1 to 100):

87	48	61	32	55	99
----	----	----	----	----	----

Adapted from Rumiati (2010, pp. 217-218). Some formulated with guide lines provided in Wright, Martland and Stafford (2006:20, cited in Wright, 2013, p. 31) and Wright, Martland and Stafford (2000, pp. 24-30).

-----*Tamaliza zikomo kwambiri. Pano ukhoza kubwerera ku kalasi. Chonde usakauze mnzako wina aliyense zimene ndimakufunsazi* -----

Appendix G: Oral assessment interview guide for learners (English version)

Name of learner: _____	Age: _____	Sex: _____
Standard: _____	Date of Interview: __/__/__	
Name of Class Teacher: _____	Time: _____	

Consent

Before we start, I want to tell you my name. My name is _____ and I am a teacher.

- I want to know how children learn mathematics. You were chosen purposively. I would like to request you to help in this. But you do not have to take part if you do not want to.
- We are going to play some counting games and some number games.
- Using this stopwatch, I will see how long it takes you to count.
- This is NOT a test and you will NOT be graded on it for school.
- I will also ask you questions about your family, like what language your family uses at home.
- I will NOT write down your name so no one will know these are your answers.
- Once again, you do not have to take part in this if you do not want to. Once we begin, if you do not want to answer a question, that's all right. Okay, are you ready to start?

Was verbal consent obtained?

Yes ☐ No ☐

If child does not give consent: do not continue with the assessment. Thank the child for his or her time and move on to the next child. If the child gives consent continue here:

Background Information

7. Did you go to any nursery/pre-school: _____
8. What language do you mostly speak at home? _____

9. Does anyone help you with school work at home? _____
10. Do you have reading books or magazines at home? _____
11. Do you watch TV? (If yes) Where? _____
12. Do you listen to radio? (If yes) Where? _____
-

1.

Task 1: Oral counting

Materials: stopwatch

Stop rule: stop the child if child makes an error while counting or at the end of a minute

- I want you to count for me. I will tell you when to begin and when to stop. Count for me from one to as high as you can count. Are you ready? **Okay begin, one...**
- Watch me count. **One, two, three... ten.** Okay, just like me, I want you to count as high as you can. Okay begin, **one...**

Time on the stopwatch: _____

Last number child said correctly: _____

2.

Task 2: Counting: One-to-one correspondence counting

Materials: Sheet “A” and stopwatch

- Here are some more circles. I want you to point and count these circles for me.
- Start here and count the circles.

How many circles did the child count: _____

- If the child does not say the number after counting the circles say: **How many circles are there?**

Number of circles child says there are: _____

Time on the stopwatch: _____

3.

Task 3: Number identification item - Exercise one
--

Materials: Sheet “B1” and stopwatch

Stop rule: Stop the child if s/he makes 4 errors one right after the other or if time on the stopwatch (30 seconds) runs out.

- Here are some numbers. I want you to point to each number and tell me what the number is. Point to first number and say: Start here.
- If a child stops on a number for 5 seconds – tell the child what the number is and point to the next number and say: What number is this?

				Number correct per row
6	1	19	4	
10	16	3	9	
15	12	7	13	

Child score (overall total correct): ____/12

Time on the stopwatch: _____

- If child gets four number word errors in a row, discontinue and go to Task 4.

Task 3: Number identification task – Exercise two
--

Materials: Sheet “B2” and stopwatch

Stop rule: Stop the child if s/he makes 4 errors one right after the other or if time on the stopwatch (30 seconds) runs out.

- Here are some more numbers. I want you to point to each number and tell me what the number is. Point to first number and say: **Start here.**
- If a child stops on a number for 5 seconds – tell the child what the number is and point to the next number and say: **What number is this?**

				Number correct per row
6	8	18	27	
5	31	47	58	
73	85	99	36	
50	59	73	91	
38	61	76	15	

Child score (overall total correct): ____/20

Time on the stopwatch: _____

4. **Task 4: Word problems**

Materials: Paper, pencils and counters

Stop rule: Stop the child if s/he gets both question 1 and question 2 incorrect

Instruction: Paper, pencils and/or counters allowed if child wants to use them

- I have some problems that I am going to ask you to solve for me. Here are some things to help you count. You can use them if you want to, but you don't have to. Listen very carefully. If you need me to, I will repeat the question for you. Okay, let's get started.

Practice item: Say: **John had 2 sweets. Sarah had 3 sweets. How many sweets did they have altogether?**

Question 1

Jane had 4 biscuits. Her father gave her 2 more biscuits. How many biscuits does Jane have altogether?

Question 2

Chisomo had 10 mangoes. He gave 5 mangoes to Katie. How many mangoes does he have left?

Question 3

There was a group of children playing a game. 2 were girls and 6 were boys. How many children were playing altogether?

Question 4

There are 9 children walking to school. 6 are boys and the rest are girls. How many girls are walking to school?

5.

Task 5: Addition/subtraction problems
--

Materials: Sheets “C1” and “C2”, counters, and stopwatch

Stop rule: Stop the child from continuing if s/he gets 2 errors one right after the other.

c) Addition problems

Instruction: Child can use paper, pencil and/or counters

1. How much is 2 and 3 altogether?
2. How much is 3 and 4 altogether?
3. How much is $7 + 3$ altogether?
4. How much is $9 + 5$ altogether?
5. How much is $8 + 7$ altogether?
6. How much is 13 and 12?

d) Subtraction problems

- **Now I am going to show you some take-away problems.**

1. How much is 5 take away 2?
2. How much is 7 take away 4?
3. How much is 9 take away 6?
4. How much is 15 take away 4?
5. How much is 23 take away 7?
6. How much is 25 take away 13?

Adapted from Malawi Early Grade Mathematics (EGMA Malawi) National Baseline Report (2010) and United States Agency for International Development (USAID) Early Grade Mathematics EDDATA II (2008).

6. Task 6: Numeral identification

I will show some numerals (random order) and ask you to tell me what number each numeral is.

Level 1 (Numerals to 10):

9	1	4	0	7	5
---	---	---	---	---	---

Level 2 (Numerals to 20)

13	10	19	12	15	20
----	----	----	----	----	----

Level 3 (Numerals to 100):

23	77	56	98	100	41
----	----	----	----	-----	----

7. Task 7: Number word sequences

(c) Forward Number Word Sequence

Procedure: Please count from *** until I tell you to stop.

Level 0: (Emergent):

1 (to 10)

Level 1: (Initial: 1 to 10):

1 (to 10)

Level 2: (Intermediate: 1 to 10):

1 (to 10)

Level 3: (Facile: 1 to 10):

1 (to 10)

Level 4: (Facile: 1 to 30):

14 (to 26)

Level 5: (Facile: 1 to 100):

49 (to 67)

81(to 93)

(d) Backward Number Word Sequence

Procedure: Please count backwards from *** until I tell you to stop

Level 0: (Emergent):

10 (to 1)

Level 1: (Initial: 1 to 10):

5 (to 1)

9 (to 4)

Level 2: (Intermediate: 1 to 10):

5 (to 1)

9 (to 4)

Level 3: (Facile: 1 to 10):

5 (to 1)

9 (to 4)

Level 4: (Facile: 1 to 30):

17 (to 13)

25 (to 19)

Level 5: (Facile: 1 to 100):

83 (to 74)

61 (to 52)

8.

Task 8: Number word after and number word before

(c) Number Word After

Now, I'll say a number (randomly) and you tell me what number comes after it.

Level 0: (Emergent):

7 5 0 4 9 1

Level 1: (Initial: 1 to 10):

7 5 0 4 9 1

Level 2: (Intermediate: 1 to 10):	7 5 0 4 9 1
Level 3: (Facile: 1 to 10):	7 5 0 4 9 1
Level 4: (Facile: 1 to 30):	13 29 17 21 24 12
Level 5: (Facile: 1 to 100):	59 77 82 44 99 31

(d) Number Word Before

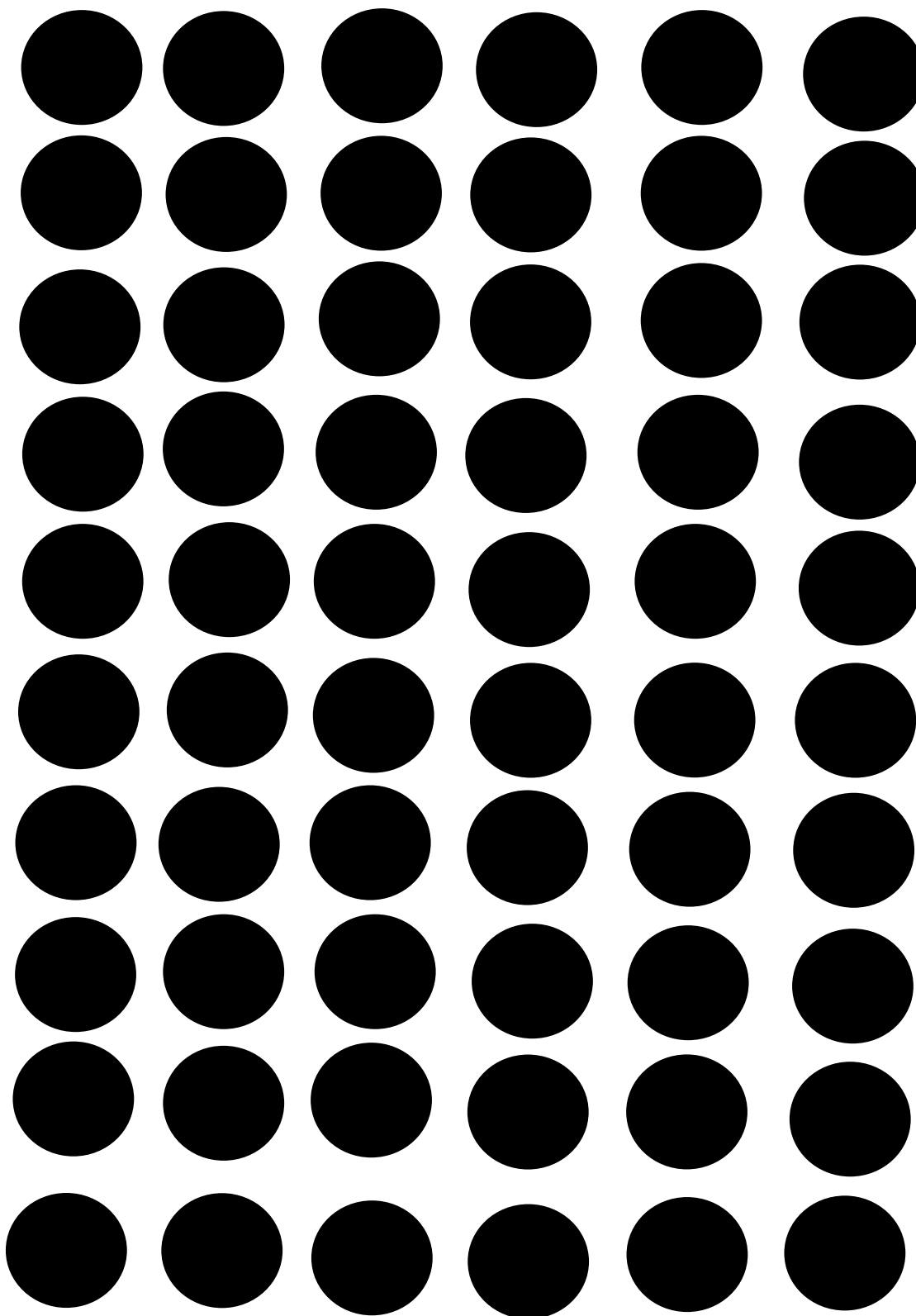
I'll say a number and you tell me what number comes just before it.

Level 0: (Emergent):	2 7 5 4 9
Level 1: (Initial: 1 to 10):	2 7 5 4 9
Level 2: (Intermediate: 1 to 10):	2 7 5 4 9
Level 3: (Facile: 1 and 10):	2 7 5 4 9
Level 4: (Facile: 1 to 30):	26 18 11 23 14 20
Level 5: (Facile: 1 to 100):	87 48 61 32 55 99

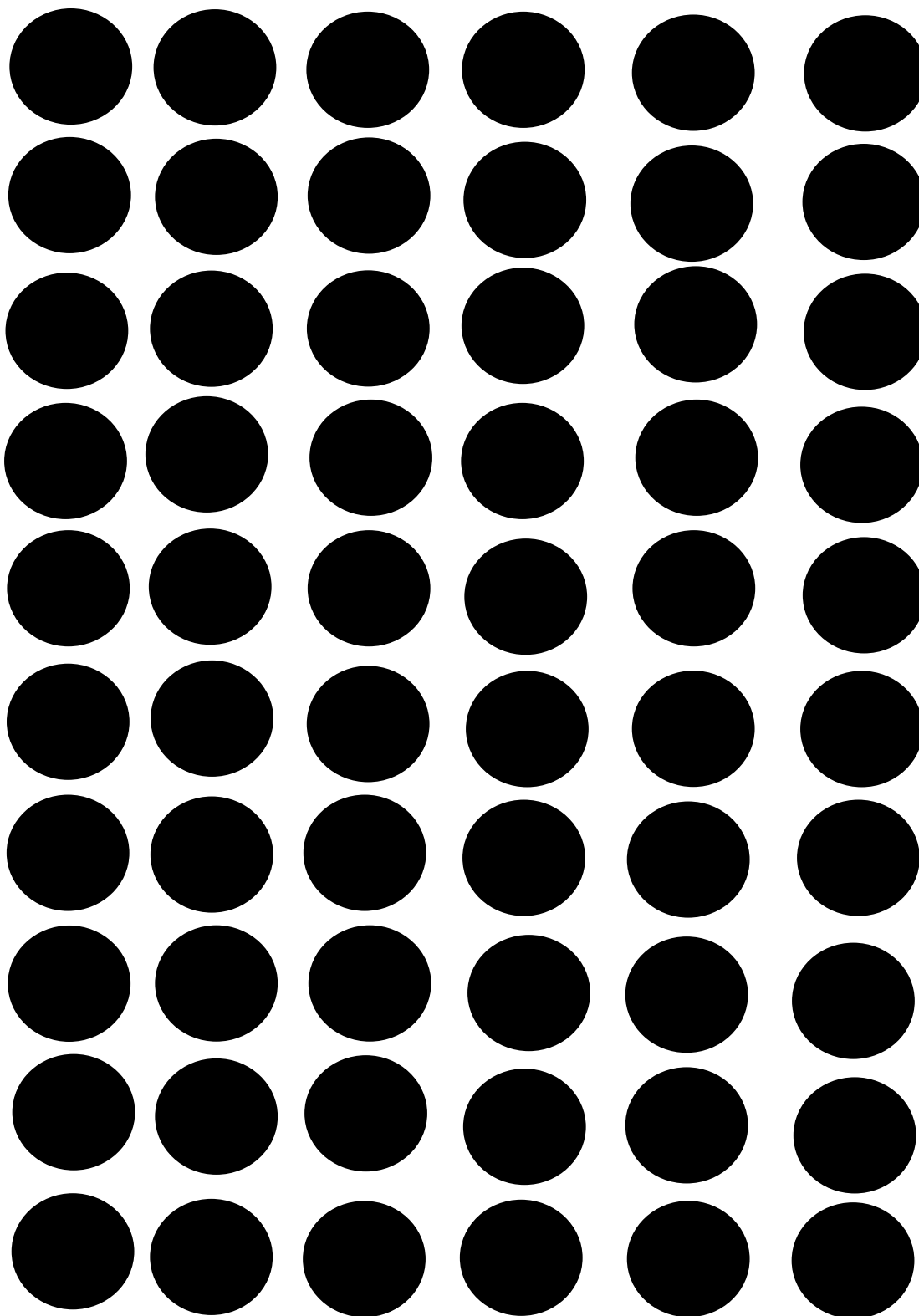
Adapted from Rumiati (2010, pp. 217-218). Some formulated with guide lines provided in Wright, Martland and Stafford (2006:20, cited in Wright, 2013, p. 31) and Wright, Martland and Stafford (2000, pp. 24-30).

-----*We have finished. Thank you very much. Now you can go back to class----*

SHEET A (1)



SHEET A (2)



SHEET B (1)

6	1	19	4
10	16	3	9
15	12	7	13

SHEET B (2)

6	8	18	27
5	31	47	58
73	85	99	36
50	59	37	91
38	61	76	15

SHEET C (1)

$2 + 3 = \underline{\quad}$
$3 + 4 = \underline{\quad}$
$7 + 3 = \underline{\quad}$
$9 + 5 = \underline{\quad}$
$8 + 7 = \underline{\quad}$
$13 + 12 = \underline{\quad}$

SHEET C (2)

$5 - 2 = \underline{\quad}$
$7 - 4 = \underline{\quad}$
$9 - 6 = \underline{\quad}$
$15 - 4 = \underline{\quad}$
$23 - 7 = \underline{\quad}$
$25 - 13 = \underline{\quad}$

Appendix H: Lesson observation guide**School Name:** -----**Date** -----**Teacher's Name:** -----**Standard:** -----**Time:** -----**Number of learners**-----**Learners' Age Range:** -----

	Question	Observations
1	What types of resources are used in the teaching and learning of number concepts (counting, addition and subtraction)?	
2	Where are the resources placed in the classroom?	
3	Who is using the resources?	
4	How are the resources used during lessons?	
5	What methods does the teacher use to teach the number concepts under study?	
6	What activities are learners engaged in during lessons?	
7	What strategies and/or skills do learners use to count, add and subtract numbers?	
8	What types of questions does the teacher ask?	
9	What kinds of explanations does the teacher give?	

Appendix I: Document analysis guide for teachers

	Teaching /learning strategies	Teaching/ learning resources	Teaching/ learning activities/experiences
Schemes and records of work			
Lesson plans			
Classroom wall charts			

Appendix J: Document analysis guide for learners

	Learning strategies	Learning resources	Learning resources
Exercise books			